

THE PROGRESS OF
ARITHMETIC

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DAVID EUGENE SMITH

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THE PROGRESS OF ARITHMETIC IN THE LAST QUARTER OF A CENTURY

BY

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PREFACE

This book is addressed to the practical teacher. It *Preface* is written with the dominant note of congratulation for worthy achievement. It seeks for the causes of progress, and it finds them in the spirit of the schools, in the discernment of the teacher in the search for the best, and in the confidence of the people who have such just pride in what the leaders in education have done and are doing and have the will to do in the future. It exploits no special method and no narrow curriculum, but it registers the progress of the past quarter of a century and it records the present status of the oldest and the most important of all the sciences—the science of elementary arithmetic.

We sometimes hear it remarked that it would have been wonderful to live in the Golden Age of Athens, when the Acropolis was the center of the intellectual, the artistic, and the political world, or in Venice when that city “held the gorgeous East” in her commercial power, or in Florence under the glorious dominion of the Medici, or at any one of a dozen times when the world was making brilliant progress; and yet we are living in just such a period today, only that the progress is more rapid and the brilliancy is more pronounced.

It is not merely in the establishing of democracies, in the daily record of marvelous inventions, and in the spread of education among all people that we are advancing, but in the tools of the workman, whether in the

Preface great manual industries or in the more important industries of the intellect. This little essay considers a single phase of this advance, but by no means the least important one,—the remarkable growth in preparing the tools of arithmetic. It sets forth the basic principles in the making of textbooks in this science, and it shows how these principles have developed in the first quarter of the twentieth century, sometimes against opposition from within the educational group itself, but with the best types of textbooks always maintaining a high standard of scholarship and of usability in the greatest of all laboratories,—the schools themselves.

No attempt has been made in this essay to list the methods either of the past or of the present, or to dwell at any length upon the teachings of either the mathematician or the psychologist. The purpose has simply been to show wherein our arithmetics have made a decided advance during the last generation, in substance, in spirit, and in mechanical excellence, and to call attention to certain typical failures that should not have been made in the past and that are without the slightest excuse in the present. No effort has been made to decide among special modes of approach to excellence in textbook-making, but attention has been called to the great principles which experience, theory, and common sense lay down as necessary to success in this field of educational activity. It is hoped that some of these results of a long study of the matter may reveal the progress that we have made and may set forth some of the many problems that confront the writer and the manufacturer of usable textbooks.

DAVID EUGENE SMITH

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THE PROGRESS OF ARITHMETIC IN THE LAST QUARTER OF A CENTURY

I

PROGRESS IN PURPOSE AND IN TOPICS

1. LET US TALK IT OVER

This little essay is a talk with you,—a personal talk with you the teacher, with you who have done the work, with you who are doing the work today, and with you who will carry it on in the days and years to come. It is an attempt to measure some of the gratifying progress that has been made in the teaching of one of the “Three R’s” (the basic group in all education), an attempt to “get together” with the teachers of the country and see what they have accomplished, often under difficult circumstances but always with an earnestness, a zeal, and an optimism that have made possible the success of the movement for better courses and for better tools with which to work. It was about twenty-five years ago that the movement of weighing values in arithmetic seriously began in this country. It was then that works of promise upon the teaching of the subject began to appear, and it was only a little later that the textbooks first attacked successfully the unreal topics and problems of the past and set a standard for the real ones of

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the present and the future. Since that movement began there has been a veritable revolution in the subject matter, in its arrangement, in the spirit with which it is presented, and in the textbooks in which the work is set forth. For example, it is only about ten or fifteen years ago that the question of the proper size of type for children of various ages began to be scientifically studied with respect to the pages of our arithmetics, with the result that today the eyestrain upon the pupils using the best of our modern textbooks has been reduced to a minimum. These and dozens of other improvements have been possible only through your help,—the help of the great mass of teachers, and particularly of those who saw the large features in the problem of presenting arithmetic to children, and who, declining to be drawn aside by ephemeral doctrine, have selected only such novelties as were backed by the common sense that comes from native endowment and from actual experience with children.

This, then, is simply a kind of heart-to-heart talk with those who are doing the real work,—an examination of the nature of the progress made and of the causes of the success that has been achieved. To a less degree it is also a study of types of failures and of the causes that have made these failures inevitable; but it is chiefly a study of success, for it is by observing success that the world really progresses and gains confidence that it will “carry on” until the goal is attained.

No one would think of denying this notable progress in the last twenty-five years. The projecting of the life element into the problem material was the opening wedge for the far-reaching reforms in approach and in organization. Hand in hand the textbook writer and the teacher have steadily advanced. They have been the two constant factors in determining the success of

the movement, the author laying out the road and the teacher paving it and demonstrating its value to the world. The chief credit for this realization "of things hoped for" a generation ago, however, is due to the teacher. It is she who has determined and who will always determine real values in the educational world. It is always a case of her common sense against unusable theories and against experiments that are foredoomed to failure. She has been little disturbed by the preaching of false prophets, and her intuition has rarely failed to lead her to accept all suggestions that make for better results in the classroom. She has had to bear much unjust criticism, often from those who themselves have been failures as practical teachers, but our country has reached its present position because of her skill and common sense and fidelity. That arithmetic is taught better today than ever before is due largely to her interest and zeal in applying the methods that experience has shown to be the most efficient.

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It is now proposed to review briefly for such teachers a few of the achievements of this period and to point out the direction in which further advance will doubtless be made.

To the great majority in the teaching profession, to those who are inspired with confidence in the present as well as the future, this brief story of the progress of arithmetic thus far in the twentieth century will, it is hoped, be of interest and value, encouraging them in their efforts to make the world a better one in which to live and labor.

2. TOPICS OF THE PAST

First, let us consider a few of the topics found in the arithmetics of twenty-five years ago and some of those found at the present time, making our comparison by

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the ever-effective device of parallel columns and selecting only a limited number from each period, but naturally those which by contrast best illustrate the advance that has been made. By so doing we shall also be able to anticipate in a general way our future progress. The topics have been selected from some of the most prominent books of each period under discussion and are as follows :

<i>Then</i>	<i>Now</i>
Arithmetic of special and unusual occupations. For example, alligation equation of payments arbitrated exchange partnership involving time true discount general average tax collectors' commissions marine insurance partial payments measurement of hogsheads, granaries, and cisterns	Arithmetic of the daily life of the people. For example, arithmetic of the home a simple bank account the check book arithmetic of the store organization of common corporations cost of production and overhead charges transportation the common industries farm problems of today
Obsolete processes. For example, greatest common divisor and least common multiple of large numbers	Short methods. For example, in making change in checking bills in common multiplication
Work with long and unusual fractions	Fractions limited to those of everyday life
Arithmetic progression	Thrift and savings
Geometric progression	Safe types of investment
Cube root	Percentage related more closely to decimals
Present worth : at simple interest at compound interest	Decimals related more closely to U. S. money
Troy weight	Graphs
Extensive work in compound numbers of unusual types	Compound numbers limited to a few really useful types of work
Gregorian and Julian calendars	Our duty to the government
Proportion as a means of solving commercial problems	Government expenses : city, state, national necessity for thrift
Ratio without applications	Systematic reviews of principles but with new problems
No reviews except by going over the same work	Minimum essentials emphasized
All topics of equal importance	

Such are a few of the many changes that have resulted from efforts begun about twenty years ago and carried along as rapidly as educational sentiment would permit, until the reforms were effected and the subject was brought to its present condition.

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3. PROBLEMS OF THE PAST

Not only has there been a great change for the better in the topics studied, but with our best writers there has been a marked improvement in the type of problems. Unfortunately, however, many writers have not learned that the necessary drill and the ability to analyze a problem can just as well come from exercises that represent actual conditions met by the great body of our people as from those that represent conditions that are entirely fictitious. For example, one of the recent unsuccessful efforts at reforming arithmetic concerns itself with problems on electricity that would puzzle many electricians, to say nothing of a child of thirteen. Another contemporary work, whose authors seem really to desire to improve school conditions, insists upon the use of proportion in solving easy problems that are far better solved by simple analysis. It also demands the change of metric measurements into our common system in cases where this would never be done even in a scientific experiment.

There has also been a decided advance in adapting the grade of difficulty of the problems to the mental ability of the pupils. In general this has been attended by gratifying results, although, as stated above, there have been many failures that could easily have been avoided by the exercise of a little common sense.

Although we frequently have to contend with such obstacles to progress, there is every reason for confidence when we compare the problems given in our best

books with the following fairly representative list taken from a popular arithmetic of a quarter of a century ago:

1. Find the value of $[84 - 7 \times 6 + (3 \times 5) - 3] \div 9$.

2. Divide

$\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{1}{4}$ of $7\frac{1}{2}$ by $\frac{3}{8}$ of $\frac{5}{8}$ of $\frac{1}{2}$ of $\frac{8}{5}$ of $24\frac{5}{12}$.

3. I bought 26 yards of carpet at $\$1\frac{9}{10}$ a yard, 3 curtains at $\$5\frac{3}{4}$ each, and 6 chairs at $\$1\frac{3}{4}$ each. What is my bill? (As if we ever used these common fractions of a dollar in this way!)

4. A vessel sailed from Portland, Me., for New Orleans with a cargo of 1528.375 tons of ice. On the way 94.85 tons of it melted. How much ice reached New Orleans? (The weight of the cargo of ice is given to within 2 lb.! It probably varied 1000 lb. while being stowed away.)

5. Reduce to ounces 5 T. 10 cwt. 24 lb. 8 oz.

6. From 5 lb. 7 oz. take 3 lb. 10 oz. 5 drams, 1 scruple, 15 grains.

7. Find the compound interest on \$4921.50 for 4 yr. 9 mo. 24 da. at 7%, using the table.

8. What is the present worth of \$3180.50 payable in 2 yr. 3 mo. 21 da., when money is worth $5\frac{1}{2}\%$?

9. A, B, and C formed a partnership. A put in \$3000 for 5 mo., and then increased it \$1500 for 4 mo. more. B put in \$9000 for 4 mo., and then, withdrawing half his capital, continued the remainder 3 mo. longer. C put in \$5500 for 7 mo. They gained \$3630. What was each partner's share of the gain?

10. If 5 horses eat as much as 6 cattle, and 8 horses and 12 cattle eat 12 tons of hay in 40 da., how much hay will be needed to keep 7 horses and 15 cattle 65 da.?

11. Three men bought a grindstone 20 inches in diameter. How much of the diameter must each grind off so as to share the stone equally, making no allowance for the eye?

12. A man bequeathed his property in such a way that his wife received \$7 for every \$5 received by each of his two sons and every \$4 received by each of his three daughters. If his estate was worth \$250,000, what was the sum bequeathed to each of the heirs?

13. Find the greatest common divisor of 462, 882, and 546.

14. A farmer wishes to put 336 bushels of wheat and 576 bushels of corn into the least number of bins possible of uniform size, without mixing the two kinds of grain. How many bushels must each bin hold?

15. Find the least common multiple of 2520 and 2772 and also of $11\frac{1}{3}$, $14\frac{2}{3}$, and $33\frac{1}{3}$.

16. Change 268_{te} on the duodecimal scale to the decimal scale.

17. Multiply 34₂₄ on the quinary scale by 23₄ on the same scale.

18. Take $\frac{3}{8}$ of 4 mi. from $\frac{7}{8}$ of 3 mi. 18 rd. 3 yd. 2 ft.

19. Divide 19 T. 17 cwt. 29 lb. 7 oz. by $\frac{4}{5}$.

20. Find the weight of an ivory ball 2 in. in diameter, the weight of ivory being 1825 oz. a cubic foot. (It would be interesting to see ivory sold by the the cubic foot.)

The author of one of the most popular books of twenty-five years ago, one containing hundreds of problems of this type, stated his faith in them by writing in the preface: "The unusually practical character of the problems will be discerned by a very cursory examination." Certainly only a "very cursory examination" could possibly lead to such a belief. It is the elimination of problems of this type that constitutes perhaps the most important feature in the advance that began about the time the book above referred to was at the acme of its popularity. There remains, however, much to be done to free the weaker arithmetics completely of such problems, particularly with respect to fractions.

4. THE WEIGHING OF VALUES

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The progress made in the topics treated in arithmetic during the last quarter of a century, and in the purposes and the spirit of the schools, is paralleled by the progress made in the weighing of values and in the consequent extent to which the instruction is carried.

In a certain popular textbook of about the year 1900 the first chapter, for example, begins with the distinction (wholly artificial and useless for any children who would have used the book in question) between numeration and notation. Numeration is then carried to decillions, with a statement that it could be carried farther,—as if anyone ever used names of this kind, even in astronomy and physics. Pupils are required to read numbers like 50962328396897, in which the figures are not separated into groups of three. Considerable work is given of a nature as difficult as the case of $984326805 \div 9818$, also with no separation into periods, and the other computations are extended to similar unreasonable limits.

Not content with carrying the work with integers to unwarranted extremes (not as a pastime, which might have been excused, but as a task), most of the textbooks of the same period failed as seriously in the weighing of values with respect to fractions. One of them gives sixty problems in the reduction of fractions, as difficult in every case as the reduction of $\frac{7\frac{2}{3}}{8\frac{1}{4}}$ to lowest terms. Twenty-five cases are given in reducing mixed numbers to improper fractions, the average of difficulty being represented by cases like $537\frac{5}{8}\frac{5}{1}$, and an even greater number of cases are given requiring the reduction of fractions like $\frac{7\frac{7}{8}\frac{9}{3}}{7}$ to mixed numbers. The following are a few of the other requirements in fractions and serve to illustrate the very impractical nature of the work:

1. Reduce $\frac{7}{11}$, $\frac{9}{13}$, $\frac{17}{33}$, and $\frac{11}{9}$ to similar fractions having their least common denominator.

2. Add $17\frac{13}{23}$, $16\frac{17}{46}$, and $23\frac{5}{6}$.

3. Find the combined weights of five men whose respective weights are $157\frac{3}{4}$ lb., $161\frac{5}{8}$ lb., $159\frac{1}{2}$ lb., $173\frac{5}{16}$ lb., and $169\frac{5}{8}$ lb. (The knowing of weights to a single ounce seemed to the author of the book to be perfectly natural.)

4. A man walked $23\frac{2}{3}$ mi. the first day of a trip, $25\frac{3}{20}$ mi. the second, $28\frac{1}{4}$ mi. the third, and $26\frac{53}{100}$ mi. the fourth. How far did he walk in all?

5. Find the value of $8\frac{2}{7} + 5\frac{4}{9} + 9\frac{2}{3} - 3\frac{2}{11} - 3\frac{6}{7}$.

6. Find the value of $\frac{2}{19} \times 7\frac{3}{4} \times 1\frac{9}{10}$.

7. Find the value of $\frac{\frac{9}{19} \text{ of } 13\frac{7}{12}}{\frac{1}{8} \text{ of } 7\frac{5}{16}}$.

The elimination of such material and the simplification of the work in fractions in general have been among the noteworthy achievements, begun early in the period under discussion and now carried to a fairly satisfactory conclusion, by which the ordinary needs of the present everyday life are properly met.

5. COMPOUND NUMBERS

If the schools of a generation ago failed entirely to weigh properly the value of the work offered in fractions, carrying the calculations far beyond the bounds which the ordinary needs of well-educated people would naturally fix, they failed quite as completely in the work in compound numbers. The following are a few more of the problems of the period:

1. Reduce 6 mi. 37 rd. 4 yd. 3 ft. 6 in. to inches, and $\frac{5}{7}$ of a rod to yards, feet, and inches.

2. Reduce 721327 inches to miles. (The number was not even written in periods of three figures.)

3. Reduce 7 sq. mi. 17 A. 13 sq. ch. to square chains.
4. Reduce 9230 scruples to higher denominations.
5. Reduce 7 hr. 32 min. 49 sec. to seconds.
6. Find the sum of 10 mi. 172 rd. 2 yd. 2 ft. 9 in., 12 mi. 172 rd. 4 yd. 11 in., 16 mi. 74 rd. 1 yd. 2 ft. 3 in., 19 mi. 198 rd. 4 yd. 9 in., and 39 mi. 131 rd. 5 yd. 1 ft. 7 in.

It would be difficult today to find in our current books of the better class any considerable number of such illustrations of the failure to grasp the arithmetic needs of the world, even as such needs were felt when these problems were framed. The tables to be learned, the absurd numbers used, the extent to which the calculations were carried, and the dull stupidity of the whole work are, from the standpoint of the present, surprising and even unintelligible. The argument that children should be carried beyond the limits of actual need has some merit, but to carry them to these lengths is unwarranted in any case, while it gives a false idea of business customs with regard to practical measurements.

In this topic, as well as in most of the others, we have made very substantial progress with respect to the weighing of values. It is true that some such work is found in certain arithmetics of the present, some of which lay claim to being progressive, but it represents a practice which, for the most part, is happily forgotten in business life and will never be revived.

6. FURTHER EVIDENCE OF IMPROVEMENT

From such evidence as this we may safely ask the question, Has there been an improvement? The answer is an unquestionable affirmative. There is not so much of the unusual in the arithmetics of today as in those of the close of the nineteenth century, and there is more of everyday life; the computations are not so hard, and

they are more like those that the world uses; the discipline of the difficult is not so much in evidence, and the motive for study, the needs of the world, the simplicity of approach, and the interests of the pupil are given immeasurably more attention and care.

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Perhaps the most important change of all is seen in the purpose of teaching arithmetic. A quarter of a century ago it was still felt that the subject should be hard in order to be valuable, and it sometimes looked as if it did not make so much difference to the school as to what a pupil studied so long as he hated it. The old idea that this was good for the mind and soul was not at that time fully discarded. There was also prevalent the idea that as many applications of arithmetic should be introduced as the time allowed, irrespective of whether they were within the mental horizon of the pupil or within the probable needs of his life after leaving school. This view has now been changed; the purpose of teaching arithmetic has come to be recognized as the acquisition of power to calculate within the limits of the needs of the average well-informed citizen. It has also come to be recognized that the problem is primarily designed to show a need for computation, by giving applications that add to the interest in calculation and by introducing the puzzle element of problem-solving, which may add further interest. A secondary purpose of the problem is the imparting of some knowledge of economic conditions. In the first six school years the first two of these reasons hold an undisputed position; in the seventh and eighth school years the economic reason is more in evidence, and such subjects as thrift, investments, civic duties, and general commercial topics occupy the pupil's attention, with calculation and the puzzle problem playing minor rôles. In all this movement for reform there has been hearty cooperation on the part of the teachers all

through the country. They have welcomed the assistance of modern textbooks, for none knew better than they the unreality of the topics and problems and the excessive manipulation of figures that characterized the earlier works. They have contributed to the making of problems suited to the mentality of the pupils in the several grades, for none knew better than they the futility of the involved problems of a generation ago.

7. THE NEW SPIRIT

There has also grown up in the last quarter of a century a consensus of belief as to the method of presenting the subject,—not in regard to trivial details with respect to subjects like subtraction, but, as will be shown later, in relation to the general spirit of teaching. We have come to believe that a pupil in school should feel that he is living his own life naturally, with a minimum of restraint and without tasks that are unduly irksome; that he should find his way through arithmetic largely by his own spirit of curiosity; and that he should be directed in arithmetic as he would be directed in any other game,—not harshly driven, hardly even led, but proceeding with the feeling that he is being accompanied and that he is doing his share in finding the way. As Professor Dewey once said, "It is pretty generally conceded that the proper method of teaching is to present the facts and let them be worked up according to the capacity of the minds that work upon them." Some minds will find the way and appreciate the view much more fully than others, but the spirit of the modern teaching of arithmetic as of other subjects is to give everyone a fair chance. To this change of spirit the textbooks have abundantly contributed, not merely in the upper grades but also in the primers of the science.

8. DISCIPLINARY VALUES

In the last quarter of a century the theory of the formal discipline of arithmetic has had its ebb and flow, like many other educational theories. At the close of the nineteenth century it was just beginning to be seriously and extensively questioned. Then came the usual flock of late discoverers, those who asserted that "the doctrine of mental discipline had been exploded." In recent years the sober sense of the teaching profession has come more in evidence and we have been asking our leading psychologists if this assertion, so loudly made by near philosophers, is scientifically sound. The result is what common sense would naturally predict,—that in the first place it depends upon the definition of mental discipline, and that in the second place the accepted definition warrants the statement that there is a well-defined and valuable disciplinary value in the study of arithmetic. It is needless to say that no one claims that rapidity in acquiring or reciting the multiplication table is going perceptibly to increase a child's rapidity in learning or reciting the boundaries of Wisconsin or Oklahoma. No one believes that accuracy in multiplication is a power that will be so transferred as to make a child perceptibly more accurate in map drawing. It would be absurd to assert that a child who comes to like arithmetic would thereby like his mother any better, or would like any more ardently to go to bed or to get up. All such suggestions of the "transfer of power" are of course ridiculous, and no one makes or ever has made any such claims for mental discipline on the part of arithmetic. If anyone is tempted to believe that this theory of mental discipline has been exploded, the only true reply is that no one ever asserted its existence, except those who wished thereby to destroy a fiction.

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The claim has been made, however, that arithmetic strengthens the reasoning powers, and there seems to be no doubt that this assertion was made many years ago with unwarranted emphasis. Everyone knows of cases where a person reasons well in arithmetic and not in other subjects. This is probably due to the fact that he has a greater interest in mathematics than in other branches of study, and hence gives less attention to the latter. At any rate, it seems clear that the claim was formerly exaggerated, although the efforts of the psychologists to weigh the value of arithmetic, or of any other subject of study, have not progressed far enough to produce results that can be described as other than somewhat crude. It is impossible to weigh a child's mind before a certain subject has been studied, and again after it has been completed, even with our carefully selected control groups, and it is equally futile to say that the subject has added or subtracted so much of mental power. We must, after all, fall back upon our own experience, with such help as experimental psychology can render, and by the proper establishing of control groups the psychologist is making good progress. The general contention of the best workers in this field to-day seems to be that there is a disciplinary value in arithmetic in addition to its immediate usefulness in the solving of the problems of daily life, the power derived from its study being capable of transfer to a considerable group of related subjects.

In connection with this matter of the transfer of power derived from the study of arithmetic to other lines, it is proper¹ to set forth more clearly some of the objects which the experience of the period under con-

¹A few passages in this connection, and occasionally on subsequent pages, are taken, with modifications, from the author's *Teaching of Arithmetic* (Ginn and Company).

sideration has shown may be sought on the culture side of arithmetic. In the first place, we seek an absolute accuracy of operation, which differs from the kind of accuracy that we seek in science or linguistics or music. The fact that we have, in thousands of problems, sought a result so exact as to stand every test leads us to set a higher standard of accuracy in all lines than we could have set without it. In other words, here is a region of natural transfer of the power derived from the study of arithmetic. This might justify the introduction of any part of theoretical arithmetic for which the pupil is mentally ready. It is one reason why cube root was formerly studied, when pupils were more mature than they are now, and in the same way it has in the past justified series (progressions) and a more elaborate treatment of primes than any business need would warrant. Here, then, is a reason for teaching arithmetic (although not with the topics just mentioned) that is above and beyond the merely practical need of the present moment.

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A similar and related reason appears in the fact that mathematics in general, and arithmetic in particular, requires a helpful form of analysis that does not stand out so clearly in other studies. "I can prove this if I can prove that; I can prove that if I can prove a third thing; but I can prove that third thing; hence I see my way to proving the first." This is the analytic form that has come down to us from Plato's time. It is more evident in geometry, but is essentially the reasoning of arithmetic as well. "I can find the cost of $2\frac{3}{4}$ yd. if I can find the cost of 1 yd.; but I know the cost of $6\frac{1}{2}$ yd., so I can find the cost of 1 yd.; hence I can solve my problem" is the unworded line of the child's analysis. Such a training, unconsciously received and often unconsciously given, is valuable in every problem we meet, leading us to exclude the nonessential and hold with tenacity to a

definite line of argument. Its power of transfer to non-arithmetical fields seems no longer to be questioned, even if it ever was, by psychologists of high rank.

Furthermore, in mathematics in general, and in arithmetic in particular, we set down our results with a clearness and terseness of expression that is not found in other subjects. It is difficult to weigh the influence of a habit thus acquired. It may not show itself today or tomorrow; but when the need for it arises, possibly long after arithmetic as a study has been laid aside, it may again appear. The assertion that a statement is "mathematically exact" is not without meaning, and the habit, acquired under a helpful, sympathetic, inspiring teacher, of setting forth the work in arithmetic neatly, clearly, and with no superfluous labor is one of those mental acquisitions that may easily transfer to the ordinary work of practical life.

These three phases of the culture side of arithmetic—the side of mental discipline—will then suffice for our present purpose, which is to show that such a side exists: (1) the contact with absolute truth; (2) the acquisition of helpful forms of analytic reasoning; (3) the acquisition of certain habits that transfer to related fields of work. With respect to all three it may be said that our psychologists are more sympathetic today than they were a few years ago. That these exhaust the list of values of arithmetic from the culture standpoint must not, however, be inferred.

In the modern view of the theory of mental discipline the better class of textbooks on arithmetic have heartily acquiesced, eliminating that which had for its sole purpose merely a hard and deadening discipline, and emphasizing those methods which recognize the teachings of the real leaders in psychological research.

9. PROGRESS IN THE MATTER OF DRILL

One other feature of progress may be mentioned in this chapter, namely, the healthy tendency in the last few years to eliminate what we may call overdrill, meaning thereby the drill upon unnecessary phases of computation, often to the neglect of the necessary drill upon the number combinations needed in daily life.

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In the early years of the century there was a tendency on the part of many teachers to feel that worthy results were attained when children had been drilled to unnecessary facility in one line or another. It is not difficult to train children to add two columns of figures at a time, or to multiply by cross multiplication, or to distinguish between seven and eight circles at a glance, or to display various other forms of arithmetic ability analogous to the acrobatical feats sometimes allowed in a gymnasium. Such activities have some little value as games, and they attract attention to a school on the part of casual visitors; but in recent years teachers have generally come seriously to doubt whether the pupil derives any real benefit from this kind of work.

Thus it happens that the American arithmetic of to-day is a fair compromise between the practical without theory and the theoretical without practice. To keep this balance true has come to be felt to be one of the missions of the teacher. The tendency is to obtain the mental discipline of arithmetic from problems that are practical, and that this tendency is a healthy one there seems to be no room for reasonable doubt.

Such are a few evidences of the general progress in the appreciation of the purposes and in the evolution of the topics of arithmetic in the past quarter of a century. We shall now consider the progress made in the teaching art in the same period.

II

PROGRESS IN THE TEACHING ART

1. PROGRESS IN TEACHING NOT SO RAPID

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The progress in the topics and in the types of problems in arithmetic in the last quarter of a century has been definite and unquestionable, but the progress in the art of teaching the subject has naturally and necessarily been less noticeable. The reason is that business conditions change rapidly in these days of new discoveries, new inventions, and new views of economics and government, while the human mind, our powers of reasoning, our rate of intellectual growth, and our moral nature have changed but little since the time of the Greeks or, indeed, of their early ancestors.

2. THE GREAT CHANGE

One change, however, has become more and more evident in the last two or three decades, and that is our sympathy with childhood,—not our affection, probably not always our good judgment, but certainly our sympathy with the child in school. The severe discipline of two generations ago had begun to relax at the close of the nineteenth century, and at the present time it has become very much less pronounced in the better type of school. The pupil has come to live the child life more freely instead of trying to live the adult life that the world not long ago sought to impose upon him. We

have still a long way to go to reach the goal, and we run continued risk of so reducing the mental food supply as to make education a poor affair and one that requires so little effort as to have neither interest nor value. On the whole, however, the average elementary pupil gets much more joy out of his school life than his parents did out of theirs, and his general range of knowledge is rather better than theirs was at the same age. It is also probable that his powers of computation in those ordinary problems of life that he is capable of understanding do not suffer by the same comparison. In spite of the easy fashion to deny this assertion, there seems no reason to think that it is not perfectly true.

All this has been a distinct gain. It has not come from any of the "methods" that loomed up so large in the eyes of the teachers of a generation ago, nor has it come to any great extent from the results of psychological studies; it has come largely from the use of plain common sense in adapting arithmetic to this new view of education,—that of letting the child live as natural a life as possible while in school,—and in adapting the work in arithmetic to his mental powers at each stage of his growth. We still have the mechanical, lock-step method of advancing from class to class, and we still have the pupil of brilliant intellect chained altogether too often to the one whose mentality is of low grade, and this is a difficulty commanding the careful study of our best educational leaders; but, on the whole, progress has been made. It has been a quarter of a century of experimentation in the effort to find in arithmetic the things most worth teaching, and then of adapting these things to the various stages of mental growth of the pupil. To this discovery and use the textbooks of the country have contributed in no small degree.

3. FAILURES OF CERTAIN EXPERIMENTS

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Among the experiments there have naturally been more failures than successes, since it is much easier to invent a machine that will not work than to get one that accomplishes what was hoped. From the long list of failures and of barren ideas a few may be mentioned as types.

For example, there was, for a time, the idea that no arithmetic should be taught in the first or even in the second grade. This notion arose from the fact that the schools were making the work unreal and too formal. As soon as the child is considered instead of the subject, however, the situation is different, for the child needs counting and delights in number games in the first grade quite as much as he needs and delights in anything else that he studies. A compromise was at one time suggested whereby children in the first grade should not study arithmetic "as such," but should absorb it incidentally,—an expectation which might with equal reason be held with respect to any other subject. The trouble was found to be that most teachers assumed that incidental teaching meant no teaching at all, and hence no effort was made to lead up to the knowledge that the child is perfectly able and willing to receive. As a result the practical courses of study of today generally include some work in arithmetic as soon as the child enters the school, although not of the dull and formal nature of a generation ago.

Among the various experiments that have failed is the one based upon the idea that no tables such as those of multiplication should be given. This arose from the fear (hardly from the fact) that children would be led to run through the table mentally whenever they wished to find the result of 7×8 , for example. The danger was

never a real one with even a mediocre teacher, for it is the universal custom of successful books to require a large amount of random drill in the addition and multiplication combinations; but at one time it frightened a certain number of teachers. As a matter of fact the multiplication table is taught largely for the purpose of allowing the pupil to build up his own number combinations, after the manner stated on page 42; it adds to his interest, to his insight into the meaning of the process, and to the increased probability that he will learn the various combinations equally well. The old plan, once in vogue, of having the children sing the multiplication table might have had some dangers, although it was probably a harmless recreation; but in the modern treatment of the tables the advantages greatly outweigh the disadvantages.

Another experiment that has not resulted as happily as its advocates hoped is seen in the dramatization of certain topics after childhood's age of vivid dramatization is past. This is seen in the great amount of time occasionally wasted in the establishing of a bank, a tax-collector's office, a clearing house, a stock exchange, and the like, with officers elected from the class. All such play can be made interesting, and it would be commendable if the object were worth the while. There is no particular reason, however, why any considerable amount of time of a class in arithmetic should be given over to "playing bank," except so far as is necessary to understand the depositing and the checking out of funds. The classes in grades seven and eight have tended, in some schools, to become groups for the study of business customs merely rather than for the acquiring of power in calculation. While this arrangement is a justifiable one in general, it is not one to be unduly exaggerated by assuming that arithmetic is a subject in which chil-

dren are to be trained in the methods of electing trustees of corporations, in the duties of the vice president of a bank, in the detailed machinery of a clearing house, or in the duties of a stockbroker. It is because of the extremes to which this dramatization has gone that the most successful teachers are abandoning a great deal of this work in the upper grades. The schools should look with disfavor upon any plan of unduly using time in teaching a subject that the average well-educated citizen will acquire in a few minutes when he meets the need for it. This is especially true when the subject is real to the child only as the teacher, often by much labor, forces it upon him. All such work becomes part of the unfortunate habit, in certain schools, of merely "marking time" in the upper classes so far as any growth in arithmetic powers is concerned.

Another one of the unsuccessful experiments, not limited, however, to the last quarter of a century, is the effort to force algebra into arithmetic in cases where it is of no service. There are certain problems in arithmetic known as inverse problems. For example, the problem of finding the interest on \$150 for two years at 6% may be called a direct problem, requiring merely the direct application of the definition of interest, $2 \times 6\% \times \$150$ being equal to \$18. If, however, we wish to find the time in which \$150 at 6% will produce \$18, we have a kind of indirect or inverse process by which we find the number of years to be two. In such cases we may evidently make use of algebra, although this is not at all necessary. Such a problem is itself rare, and when it arises it is about as easily solved by common-sense analysis as by algebra. There are, in fact, few practical problems in elementary arithmetic in which algebra is needed, although it can, of course, be injected into the solutions. For this reason we have, in the last few years,

made less use of the subject in solving arithmetic problems than was the case a quarter of a century ago.

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Among the other unsuccessful experiments have been those that led to excesses in such subjects as graphs and measuring, and to an extreme belief in the value of a visualization process after its need has passed. As to the value of the graph itself there can be no question, but there is a very serious question as to the value of studying all possible kinds of graphs and as to spending an unreasonable amount of time on the easy ones that are ordinarily met. The work is simple, it is interesting, and it is valuable up to a certain point; but it can easily run to excess and may readily consume time that may better be devoted to other topics. This is equally true with the work in measuring; the subject is of undoubted value within certain limits, but when it degenerates into measuring merely for the sake of doing so, with no other end in view, it becomes wearisome in its time-consuming efforts and absurd in its pretensions. We find the same condition when we attempt to visualize everything we do in connection, for example, with fractions, forgetting that there is a value in representing fractions by lines or surfaces or solids, but that the value becomes negative if we continue the process after it has served its purpose.

Such are a few of the failures or partial failures that have resulted from various well-meant experiments in recent years; but they are typical. Others will be found as we proceed. To extend the list, one has only to read the current educational literature, particularly in the periodicals intended for elementary teachers, to see how rarely have the hopes of the experimenters succeeded. With the superior opportunities offered by some of our better-equipped schools of today the outlook is more promising.

Another failure in improving the art of presenting arithmetic in our schools is seen in the fact that we have not yet overcome the habit of making the subject unnecessarily difficult for the pupils. This idea is hard to eradicate, and it is one of the curious features of textbook making that the idea is today evident chiefly in those books that are made by the general educators,—that is, by the very ones who claim to lead in reforms. In other words, the author of books of a generation ago might readily select from books of today certain problems that are as little suited to the needs or capacity of children as were those of his day. It is an interesting fact that more of these exercises are to be found in some of the books that claim to be written on the most advanced educational principles than in those which represent the work of the successful and experienced teacher. Here, for example, are a few problems from recent books that were written in whole or in part by men of considerable standing outside the domain of mathematics, and who claim to be more or less radical in efforts for reform. The exercises are intended for children of the age of thirteen, with no previous training in a junior high school or in physics, and they are supposed to be models as to difficulty, subject matter, and statement.

1. $T_c = \frac{T_F - 32}{1.8}$. (The pupil is supposed to understand, read, explain, and use the formula.)

2. The number of watts supplied by an electric current is equal to the number of amperes in the current $\times$ the number of volts. Write an equation explaining the fact stated in this sentence, if we: Let P represent the power in watts. Let I represent the electrical current in amperes. Let E represent the voltage in volts. (Punctuation as in the original, so also "voltage in volts.")

3. A boy's lung capacity increases from 1000 cu. cm. at 6 years of age to 3600 cu. cm. at 18 years of age. Express the difference in cubic inches. (The problem is not practical, inasmuch as, in such cases, the metric system is used by itself, or our common system by itself. A difficulty is raised for the mere purpose of having one.)

4. If 24 cows give 3275 lb. of milk per week, how much milk would be produced per week if 12 cows of the same average capacity were added? (The problem, which can be defended, even if unreal to most pupils, is rendered unnecessarily difficult for children of the age of thirteen by the requirement that it be solved by proportion, a method no longer necessary or desirable for this type of exercise.)

5. The diameter of a piston is 6 in. and the diameter of the piston rod is $1\frac{1}{4}$ in. The air pressure is 90 lb. to the square inch; that is, the pressure on each square inch of the lower circular face of the piston is 90 lb. Will it be able to press up against that part of the piston's surface which is occupied by the piston rod? How much will this hoist lift? (Imagine the effect upon both pupil and teacher!)

6. A rectangular tank has a capacity of 400 liters. What dimensions might such a tank have? (It would be interesting to know how the teacher would handle this indeterminate problem with its infinite number of answers.)

7. A bushel of shelled corn weighs 56 lb. Find the weight of 2 pt. (As if anyone would ever need to solve such a problem!)

8. If I mix 19 gal. of water with 23 gal. of grape juice, what per cent of the mixture is water? (Just what is the purpose of this mixture?)

Arithmetic can be made unreasonably hard or unreasonably easy. The desirable mean is the one that represents conditions which the pupil is likely to meet in actual life, and which he is capable of comprehending fully when they are presented to him in school.

4. PEDANTRY IN ARITHMETIC

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The unnecessary difficulties of arithmetic are not confined to those for the pupils; the teacher has to suffer as well; and this is seen in what may be called the pedantry of the educator. To many people the use of unnecessary but high-sounding terms—that method of impressing the youth which is known as pedantry—seems highly desirable. For a time and with certain minds such a method may have some value, but in the long run it fails, just as all sham and pretense must fail.

From the day when the first teacher had his first pupil even to the present time successful instruction has been attained only by first making the object of study seem worth the effort. Knowledge has been beaten into the heads of youth, but even then the learner deemed it worth while to avoid the beating. When the word “motivation” was coined there was not minted with it any idea that was new; it was merely that a new name had been added to a vocabulary already overstocked.

The masters of ancient times taught by parables, but they were careful to see that what they taught was easily related to daily life, and their successors even to the present time have always done the same. Relations between subjects, between the school and the world beyond the walls, and between the fictitious and the real have always been sought and emphasized by the successful teacher. To indicate this we coin the term “correlation” and then let it lapse into disuse. To indicate the relation between a stimulus, a mental activity, and a reaction we coin the term “bond relation,” but we coin with it not a single new idea, nor is any new concept set forth when we apply the name to an algebraic situation. We use the term because it impresses the unthinking; but if we give it thought ourselves, we must apologize to

these same selves for our pedantry. A technical term may be a necessity if used in the technical laboratory, but its use in a science intended to be clear to a vast array of teachers is quite another matter.

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If we can speak of "organizing our knowledge," of "organizing ideas," of "objective standards," of "motivated practice," of "arithmetical activity," of "interpretations," of "content," of "topic of activity," of "quantitative relations," of "interpreting a graphic distribution of frequencies," of "the reciprocal rule generalized," of "making amounts commensurate," and of "errorless counting and tabulating,"—if we can speak of all these, and of "gaining control of a formula," and the like, in textbooks for children or in elementary manuals for teachers, without smiling at our own pedantry, we certainly have become callous through much pedagogy. Every generation, however, has to learn anew to appreciate the words of the ancient Latin writer Ammianus Marcellinus, "The language of truth is unadorned and always simple."

Of course we excuse ourselves, however, on the plea, already suggested, that every occupation requires its own technical vocabulary. The plea is specious, but is it really honest? Do we need these terms in books intended for children who are already overburdened with school terms that they will never meet in daily life (and it is from various arithmetics that this list was compiled), and do we who write books deal fairly and honestly with those who are placed in our care by any such display of the language of the pedant? Let us have all the good things that these terms stand or pretend to stand for, but let us seek simplicity of statement rather than mere verbiage and pretense and sham learning, and especially let us be careful to do this in the textbooks themselves.

5. MISTAKEN EFFORTS AT HUMANIZING

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And, finally, a further example of the failures of the last twenty-five years may be seen in some of the extreme efforts at humanizing our arithmetic. We have made great progress in the general movement, but there are always those who grasp only the word and never the spirit. As a result we have a considerable number of teachers who succeed in so humanizing the subject as to leave only humanity and no arithmetic. Their pupils cannot compute with any facility or accuracy; they play but they do not achieve. In this they are occasionally encouraged by some textbook that seeks to carry not merely the spirit but the intellectual standards of the kindergarten up to the threshold of the high school. Their problems in the seventh and eighth grades have few computations of difficulty beyond that of the primary years, and the psychological value of drill is often wholly forgotten. Fortunately, just as the biologist finds a means to eradicate a pest that kills the plants and animals upon which we depend, so the practical teacher has come to our aid with certain arithmetic tests that tend to hold us to at least some degree of efficiency. The most serious trouble with these tests lies in the fact that they are based upon the average achievements of our pupils and therefore tend to standardize mediocrity. This defect will in due time disappear, and already the tests have abundantly proved their value.

6. CERTAIN FAILURES IN RESEARCH

Among the experiments made in the field of teaching are those which are based upon the fallacy of certain types of "research" with respect to the public demand for certain special phases of arithmetic. For example, we may ask a thousand persons to keep track of the

number of times, in a month, they use counting, addition, subtraction, and so on, and also to make a list of the fractions they use. This is an interesting thing to do and it is even profitable to know, statistically, the result. But the trouble lies in the interpretation of these results. To take a hypothetical case, suppose that we find in the newspapers of a certain part of the country that, in some particular month, the subject of multiplication is referred to more often than addition; what will it mean with respect to the relative emphasis to be placed upon the two topics in teaching? Absolutely nothing. Had the investigation been made in grocery stores the result would not have been the same; in dry-goods stores it would have been still different, and so on. One of the latest investigations, and one that was very carefully made, shows that in the computations connected with twenty-five different industries 53 per cent of the operations were in addition, 41 per cent were in multiplication, 4 per cent were in division, and only 2 per cent were in subtraction. These results are authentic, but what do they prove with respect to teaching? That we should put twenty-five times as much effort, attention, and drill upon addition as upon subtraction? The idea is absurd. Even if the man who tries to have his check book balance with the books of the bank may have fewer occasions to subtract than to add, he knows perfectly well that accuracy is just as important when the subtraction is needed as it is in the case of addition.

As a result of just such false inferences the schools have begun to weigh more carefully the erroneous conclusions drawn from statistical data, and even (though often unjustly) to challenge the value of masses of statistics which require only borrowed ability in their planning and no intelligence in their accumulation, and which often show but little talent in their interpretation.

This sort of "research" has not been carried on by trained psychologists or even by highly trained specialists in other fields. Nevertheless it represents an interesting phase in the advance from the rather stagnant conditions of a generation ago. That the psychologist has been more successful in his method of investigation is, of course, to be expected; that he has always been successful in acting upon the results of his own work is unfortunately not the case.

7. EXPERIMENTS OF PSYCHOLOGISTS

The experiments made by the psychologists in the application of their results to the subject of arithmetic have, as suggested in the preceding paragraph, failed to a greater extent than was expected a few years ago. This is seen in the lack of proper grading of problems on the part of teachers who have had psychological training, and in the interpretation that they give to certain statistical tables which they adduce. It not infrequently happens that they call attention to the fact that certain fractions are used to excess in a particular book, but they fail to see that they are used to about the same extent in the problems of daily life. Similarly, because multiplication is used twenty times as often as subtraction in a certain investigation, as in the case mentioned above, they seem to give the impression that we should teach it twenty times as carefully, or twenty times as long, or with twenty times as many exercises.

It must be said, however, that there are not nearly so many failures in obtaining reliable statistics and in sensibly interpreting them as there were a few years ago. The trained statistician has now replaced the tyro, and it is coming to be recognized that a year's course in educational statistics will not insure the world against a lack of common sense in the interpretation of results.

Probably the most disappointing of all the efforts of the psychologists is seen in their unintentional efforts to standardize parts of arithmetic that are tending to disappear, thus leading the schools to retain them. The psychologist seems to devote his efforts to showing only the best methods of teaching what is, not in finding out what ought to be. He excuses himself by saying that the latter is the duty of the educational engineer, of the efficiency expert, of the economist,—in general, of somebody else. This may be true, but the fact remains that when he investigates, in algebra, the best method of approach to the study of $\sqrt{625} \div \sqrt{10}$ as a case in the work in radicals, he thereby tends to standardize this as an important topic to be taught in an elementary class in the subject. As a matter of fact its importance is very slight, either there or in the practical applications of the science.

We meet a similar difficulty when we consider various tests in the several fields of mathematics. There is no agency that so tends to the maintenance of the traditional, and the most useless part of the traditional, as some of the modern psychological tests. In this respect they are no improvement upon the older type of examinations that were and still are set by the state, the city, the colleges, or some other extra-school authority. This need not be the case, and it is not intentional; but the fact is patent to everyone that it has often been the result and that it has as often hampered progress in the teaching of both arithmetic and algebra. The effort has often seemed to be in the direction of finding how many operations can be performed in the twenty seconds between "Go" and "Stop" rather than of discovering whether the particular operations were worth the performing or whether the speed sought was worth the seeking.

8. ASSISTANCE RENDERED BY THE PSYCHOLOGIST

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Looking now at the more successful efforts to advance the teaching art, we may first turn to the real assistance rendered by the psychologist. Where he has helped most in the teaching of arithmetic is in the prognostic tests which he has set for the purpose of determining how best to group the pupils. In this way he has helped break the chains that hold the brilliant pupils to the pace of the dull ones. In large schools, where the formation of several classes is possible, the results of these tests have been of great assistance. The basal notion underlying all prognostic tests is the measuring of the predestination of the individual. There is, of course, a well-recognized basis for Calvinism; it is only Calvinism gone mad that the world abhors, whether in theology or in education.

Two generations ago Ruskin wrote out the patent facts which the world has always accepted:

The greatness or smallness of a man is, in the most conclusive sense, determined for him at his birth, as strictly as it is determined for a fruit whether it is to be a currant or an apricot.

The simile is a poor one, but better ones will easily occur to any of us. Ruskin goes on to say:

Education, favorable circumstances, resolution, and industry can do much; in a certain sense they do *everything*; that is to say, they determine whether the poor apricot shall fall in the form of a green bead, blighted by the east wind, and be trodden under foot, or whether it shall expand into tender pride, and sweet brightness of golden velvet. But apricot out of currant,—great man out of small,—did never yet art or effort make; and, in a general way, men have their excellence nearly fixed for them when they are born. . . .

Therefore it is, that every system of teaching is false which holds forth "great art" as in any wise to be taught to students,

or even to be aimed at by them. Great art is precisely that which never was, nor will be taught; it is pre-eminently and finally the expression of the spirits of great men, so that the only wholesome teaching is that which simply endeavors to fix those characters of nobleness in the pupil's mind, of which it seems easily susceptible.

In the same way, we cannot make great arithmeticians out of all; we can simply give to all the opportunity to use their powers at the best. This is or should be the spirit of American education. Just as the psychologist has been of great assistance in this field, so he will, as he comes to realize more clearly the way in which his help can best be rendered, be of equal assistance in lines in which, as already stated, he has not offered the aid that was anticipated.

In such fields as those relating to the motive for work, to the scientific proof of the value of drill where mechanical efficiency is needed, to the purpose and the advantage of reviews that are made interesting instead of dull, to the doctrine of interest in general, to the understanding of the will, to the value of a sane interrelating of topics as opposed to a senseless attempt to mix up the unrelated, and to the cultivation of initiative through the proper use of games,—in all this the psychologist has done a great service in advancing the teaching art.

He has perhaps helped to confirm the remark of that delightful purveyor of the obvious, G. K. Chesterton, that the reforms of each generation are directed to undoing the reforms of the generation immediately preceding. By his neglect of such educational phrases as were in common use a quarter of a century ago—"concentration," "learn to do by doing," the "five formal steps," "apperception," and the like—he has helped us to see the futility of basing the art of teaching upon newly coined terms that stand only for that which everyone

knows already and can express in everyday language. If he helps us properly to weigh some of the similar names and phrases of today, some of them unfortunately coined by himself, we shall all have reason to be even more grateful.

9. TYPES OF VALUABLE SUGGESTIONS OFFERED BY PSYCHOLOGY

Psychology has come to the aid of practical teachers by offering a number of helpful suggestions and, more especially, by confirming through scientific experiment the conclusions intuitively formed as the result of the teacher's own experience. The following, relating chiefly to drill work, are types of statements to be found in recent works on educational psychology :

1. All the hundred combinations in addition must be mastered if we are to expect reasonable speed to be shown in any of the various number tests.
2. The work must be introduced gradually, first by adding 1, then by adding 2, then by adding 3, then 4, and then 5, after which the higher sums should be systematically studied in any one of several methods.
3. For purpose of drill, special attention should be given to the troublesome groups.
4. The facts of addition should be established by the aid of objects; but "habituation is more important than rationalization," as the psychologist would phrase the statement that habit is of more consequence than reason in work of this kind, and hence the pupil should reach the more abstract work as soon as possible.
5. The drill should be carried on by means of games and of speed contests, leading to the keeping of the record by each pupil and to his desire to improve upon this record in subsequent tests.

6. There should be simple problems involving all the early number facts, the work not being left abstract.

7. Adding three or more numbers should begin with small numbers, and all such work should be introduced by real problems.

8. What the psychologist speaks of as "bridging the tens," or "establishing the bonds" $4 + 7 = 11$, $14 + 7 = 21$, $24 + 7 = 31$, and so on, demands special drill. The practical teacher employs simpler language and usually speaks of this as a drill on endings, the endings of the numbers being alike. To facilitate this work it is desirable to learn to count by 2's, 3's, and so on.

9. The addition of two-figure numbers should begin without "carrying." It should then pass to the case of carrying only one ten. In the case of three-figure numbers the carrying should begin with one ten in the units' column, then one in the tens' column, and then one ten in each of these two columns. The carrying in the thousands' column should come later. The "rationalization" of the carrying process should be considered but should not be demanded from a pupil.

10. Addition of columns with more than six numbers should come later, grouping where there is any gain.

11. Although great speed is not expected, being quite foreign to the habits of most adults in performing additions, a reasonable number of speed tests in all the operations should be given, if only for the interest which the game affords.

Not one of these suggestions is at all startling; any practical teacher knows them; but they are taken substantially from a recent psychological essay that has attracted considerable attention.

In the work in multiplication and division psychology urges such speed tests as are given in all arithmetics of the practical kind, and recommends the common device of flash cards. It also finds that the "bond" between

9×8 and 72 often fails to operate when there is required such a multiplication as 9×898 . This requires the carrying drill with which pupils should become familiar in cases like $9 \times 8 + 8$, but which is unfortunately missing from many arithmetics. The value of a reasonable time limit in all these cases of drill is now generally recognized. Unfortunately some of the psychologists would include a considerable amount of such objectionable work as the following:

$$.0325 \times \$103.25$$

instead of

$$0.03\frac{1}{4} \times \$103.25,$$

the second form representing the common business practice;

$$12 \times \$\frac{5}{6},$$

which leads the nonthinking person to believe that this is a feature to be commended;

$$7\frac{1}{2} \times 11\frac{3}{4},$$

which suggests that this type of work is so valuable as to be worthy of special drill; and

$$90.08 \times \$726,$$

which has the same implication.

With respect to the difficulties in division some recent tests have shown that 30 per cent of the errors are due to a lack of knowledge of the multiplication table, which a few teachers say they would like to see abolished. The best psychologists, however, properly recommend that the subject of division should be definitely based upon the multiplication table. They show, as any good teacher knows, that one habit or ability is simply a stepping-stone to another, and so the work should proceed by easy gradations.

In the case of fractions recent tests have thrown some light upon the nature of the difficulties. In addition, as

of $\frac{1}{2}$ and $\frac{1}{4}$, over 30 per cent of the pupils who tried one test added the numerators for the numerator of the sum and the denominators for the denominator of the sum, the result appearing as $\frac{1}{2} + \frac{1}{4} = \frac{2}{6}$.

Nearly as many failed to reduce results to lowest terms. Approximately 10 per cent gave results like these:

$$\frac{3}{4} + \frac{2}{3} = \frac{6}{12},$$

$$\frac{4}{5} - \frac{1}{2} = \frac{1}{5}.$$

One great difficulty in the teaching of fractions is that little motive is offered and the work seems abstract and unreal. If introduced, as it should be, in connection with concrete objects (and particularly with measures), if the order of difficulty is carefully kept in mind, and if there is sufficient drill work, these errors will rarely occur. Moreover, if those who profess to have been students of psychology will see that the operations with fractions should not begin with addition, but rather with cases like $\frac{1}{2}$ of 4 in., they will greatly assist their pupils. If they will also help toward progress by eliminating such types as adding fourths and ninths, or sixths and fourteenths, or ninths and fifteenths (all of which appear in a recently published diagnostic test), they will serve in a good cause. In another recent piece of "ladder" work, pupils are required to solve problems like these:

$$3 \text{ min. } 10\frac{1}{4} \text{ sec.} - 93.4 \text{ sec.}$$

$$3.42 \text{ mi.} - 2\frac{3}{8} \text{ mi.}$$

$$6.78 \text{ ft.} - 2 \text{ ft. } 9 \text{ in.,}$$

all of which are inserted for the purpose of having certain difficult cases, but which have the effect of standardizing these nonsensical forms. A progressive arithmetic may be judged in part by its avoidance of these objectionable types.

The psychologists seem to have shown without any reasonable doubt that accuracy and speed are independent qualities, although it is probably also true that very slow mental action leads to absent-mindedness of a sort that makes for inaccuracy. It is possible and desirable, however, to train for 100 per cent accuracy and a fair degree of speed, even though the former is a shifting goal depending upon the time allowed in a test.

For the teaching of accuracy the subject of checks is of great importance. For acquiring speed the speed tests are valuable, although there is no particular reason for demanding unusual excellence in this particular. Very few adults compute rapidly, and very few have any need to do so, although a fair degree of speed is desirable.

The psychologists have done a service in calling attention to the desirability of a simple vocabulary in arithmetic, although those of them who have written arithmetics have usually violated the principle more frequently than the mathematical writer has ever done. The fault will not be corrected by the ordinary vocabulary tests in arithmetic; it will be remedied by actually using simple language in our textbooks and in our books for teachers.

Mention was made above of our indebtedness to the psychologist for his scientific proof of the value of drill, and a few words should be added to what was there said.

It is one of the best-established laws of the mind that constant repetition of an act leads to the formation of a habit of doing that act or of responding mechanically to the impulse which stimulates the act in question. Thus we write rapidly without stopping to think of the letters or of the spelling, because we have been drilled, or have drilled ourselves, for years to do this thing; and the stimulus "two times seven" brings the response "four-

teen" without our stopping to think, this being the result of thousands of repetitions of the process. The first printed arithmetics had no drill, and the operations were time-consuming in a period of history when time seemed not so precious as now. When these books were printed, people generally multiplied, in a case like 7×8 , on their fingers, or even on an abacus, or number frame, as it is sometimes called. We could get along without drill to-day if we would be content to be as slow in our computations as the merchants of Italy were when Columbus discovered America. If, however, we feel the need, in our present civilization, of knowing instantly the product of 7×8 when we see or hear the 7 and the 8, then we must have this product so thoroughly fixed in mind that we shall not hesitate an instant. The process must become mechanical, and the only way that psychology has found to make it so is through long-continued repetition,—a process which, for brevity, we call drill.

Certain psychologists have also called attention to a fact known to most practical teachers by experience in the classroom; namely, that pupils find much pleasure in drill with abstract numbers. This is partly due to the rhythm of number, as in counting by twos, threes, fours, and so on, and partly to the game element, to the love of achievement, and to the healthy rivalry generated by the keeping of score cards in modern tests.

What is here said about arithmetic is equally true with respect to elementary algebra. "Rationalization" and "motivation" have their place in the initial steps, but mechanical efficiency arising from systematic drill is a necessity if one is ever to use the subject practically.

In all these matters of drill the books of the present day are far superior to their predecessors, and the genesis of this superiority is easily found by searching the texts of the last few years.

10. IMPROVEMENTS IN ACQUIRING NUMBER FACTS

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One of the decided advances made in the art of teaching in recent years relates to the process of acquiring number facts, and it has been due to the combined efforts of theorists and practical teachers.

There are various ways of acquiring number facts. We may tell a child that 3 and 4 are 7, command him to learn the fact, and punish him if he does not do so. He has then a motive for exerting himself to commit the fact to memory. This was formerly the most common form of "motivation"; it was rather effective, but it was both brutal and unnecessary.

We may try the experiment of waiting until the child meets the need for knowing the sum of 3 and 4, and then either tell him what it is or else let him find it by counting objects. We occasionally hear it said that the latter plan is the better, but the one who asserts it will be troubled to explain away the fact that he himself goes to dictionaries and encyclopedias and tables for the information he needs, not attempting to discover it independently,—which shows that there is an argument on the other side. The assumption that no number facts should be taught until the child naturally feels the need for the information might easily result in his never learning thoroughly the product of 8 and 9.

Instead of either of the above extreme methods we may proceed systematically to have the child learn the important number facts, beginning with the simpler ones, and may create an artificial but seemingly real need which shall supply the motive. This is what we commonly do in the schools. We may then lead the child to add to his interest by finding out the facts for himself and arranging them in some attractive way which shall assist him in his discovery and his memorizing. This

also is the method dictated by good psychology and followed by the most successful writers and teachers. It results in such devices as the multiplication table, which insures a systematic building up of the knowledge involved, and in random drill, which avoids the danger of attempting to recite the table whenever we need such a product as that of 7 and 9. It gives a visual image of all these number relations, which is a valuable psychological asset, and it insures the impartial acquiring of all the combinations, which never comes by using only the random plan, as has occasionally been attempted.

Twenty-five years ago this independent formation of tables was generally an unsystematic and uninteresting affair. One of the most popular books of that day gave the square array of the multiplication table with no indication of the way in which it was formed. Another one gave the tables in the form

1	2	3	4	...
2	4	6	8	...
3	6	9	12	...
4	8	12	16	...
⋮	⋮	⋮	⋮	
⋮	⋮	⋮	⋮	

$$2 \times 1 = 2$$

$$2 \times 2 = 4$$

$$2 \times 3 = 6$$

$$2 \times 4 = 8$$

and so on, instead of the much more natural and psychological arrangement beginning

$$1 \times 2 = 2$$

$$2 \times 2 = 4$$

$$3 \times 2 = 6$$

But even the author of the book referred to paid no attention to the systematic building up of these relations, and but little attention to random drill.

The modern plan is to lead the child to build up the table, thus making it a creature of his own. The best device thus far suggested is the following:

				2
			2	2
		2	2	2
	2	2	2	2
$\frac{2}{2}$	$\frac{2}{4}$	$\frac{2}{6}$	$\frac{2}{8}$	$\frac{2}{10}$

and so on until ten 2's are added. By this plan the pupil realizes the relation between multiplication and addition; he simply counts by 2's (a valuable adjunct to his work in addition), and he states his result in the most rational manner possible, namely,

$$1 \times 2 = 2$$

$$2 \times 2 = 4$$

$$3 \times 2 = 6$$

$$4 \times 2 = 8$$

$$5 \times 2 = 10$$

and so on. The table is his creation, he sees its significance, he visualizes the results, and he has not neglected any products that he will need. He is now ready for the random drill which every teacher and every usable textbook gives orally and repeatedly until the process becomes mechanical, just as in the case of the writing of words or the reading of a sentence.

All this is adapted to "motivation," to "busy work," to "random drill," and to "group contests," and it is a great improvement upon most of the work of a generation past. Unfortunately some of our recent books, however, adhere to antiquated models of the tables, a few writers do not give them at all, and others give so little

drill, with so little vitality and so few applications, as to handicap the pupil in all his after life. *Progress in the Teaching Art*

If the number facts are ever to become mechanical they must be acquired early; we do not readily "react" mechanically to the "stimuli" of all such combinations as 7×8 and 9×6 if we postpone the acquisition until after the primary grades.

There has of late come into the schools the phrase "3 twos are 6," this being intended to replace the older phrase "3 times 2 are 6." It is a harmless change but not an important one. When we develop the multiplication facts by the convenient plan here shown, which is essentially that of counting by twos or of adding twos, we see that we obtain 6 if we take 2 three times, so that we have "3 times 2 are 6"; but we also see that there are 3 twos above the 6, so that "3 twos are 6." The argument is often advanced that the expression "3 twos" is shorter than the expression "3 times 2"; but "2 twos," "3 threes," and the like are not such euphonious phrases as "2 times 2" and "3 times 3." Our better class of arithmetics use both forms, and this is a wise course to pursue. The world will settle the question, not the schools. At present in all languages "2 times 2" is nearly universal, probably because it sounds better than "2 twos." Whether this form will continue no one can say, but the common sense of the world does not pay much attention to the small notions of the schools. We must have something that is really significant before the world will even give it thought.

There has also been an improvement in the recognition of the reasons for limiting the facts to be learned. Teachers often ask whether they should extend the multiplication facts or table (a word that is unduly disturbing to a small minority of teachers) as far as 12×12 .

The older books generally did so, and the modern books usually give the tables of 11 and 12 as optional, simply because certain courses of study require them. The English books give them because the British monetary system has 12 pence in a shilling, and hence the ordinary computations of business render this knowledge desirable. Our early books followed English models and retained the tables. At present, in spite of the fact that we use a foot of 12 inches, there is no real justification for memorizing the table beyond tens, which means, practically, learning only to nines.

The very early tables gave 3×4 but not 4×3 , since one result is readily and almost instantly derived from the other. During the last century writers and schools have usually given both forms, thus about doubling the size of our tables. Psychology and common sense (which at the best are pretty nearly the same, but at the worst are unfortunately very different) tell us that the "reaction" to 3×4 is not the same in time as that to 4×3 , and that therefore, for purposes of speed, both forms should be known. It is a fair question, however, as to whether the future may not decide that the gain in speed is not worth the expenditure of the time and energy necessary to acquire this mechanical efficiency. We have to consider that the computing machines (in particular, the slide rules) are still in their infancy and that it is not the business of the school to attempt to make the "lightning calculator." Our motto should rather be "absolute accuracy and a reasonable speed which shall meet the needs of the average citizen" than "speed at all hazards." For the present, however, we should remain on the safe side and teach both forms (3×4 and 4×3) equally well.

What has here been said of the facts of multiplication may be said with equal justice with respect to the facts

of addition even before a textbook is used. The tables should be built by counting, just as the tables of products are built by addition, and the random drill should have the necessary attention. Unlike the facts of multiplication, the facts of addition are usually known before the textbook is placed in the hands of the child, and hence the tabular form is usually omitted.

The arithmetics of a quarter of a century ago did not, in general, give tables of subtraction and division. It was argued that if a child knew that $3 + 4 = 7$, it was unnecessary to teach him that $7 - 3 = 4$, because he could figure that out for himself. The argument is correct if we eliminate the question of time, but it is fallacious if we wish for speed in calculation. If we decide (as we must under present conditions) to teach 8×7 as well as 7×8 , we must also decide to save time in the same way by teaching both $8 + 7$ and $7 + 8$, and also $56 \div 8$, $56 \div 7$, $15 - 8$, and $15 - 7$. Not to do so is to destroy the argument for both 8×7 and 7×8 .

Hence it comes about that our best arithmetics teach systematically all these forms, and drill upon them. If calculating machines finally become so cheap and convenient as to replace our present methods of computation, the world will little by little drop from the schools some of these relations, but that time is a long way ahead.

As to the acquiring of the necessary number facts by means of games, of simple problems of childhood, of speed-and-accuracy tests, of the pupil's self-made record of his progress, of the use of objects, and of all other devices which an interest in children has developed in the last generation, there can be no question. They all have place so long as no one of them becomes an obsession and thus drives out the others or results in attention to the device rather than to efficiency in computation.

This danger is seen in certain textbooks, where the interest in play is stimulated delightfully but the drill that is needed to fix number relations thoroughly in mind is sadly neglected.

In general, however, there has been a decided improvement in the last few years in acquiring the number facts. This improvement, like all changes for the better, has come by slow degrees,—the method that nature follows in advancing standards of living, in bettering the conditions of the people, in popularizing government, and in growth of all kinds, in art, in letters, in politics, and in education.

11. PROJECTS

Certain attempts to improve the art of teaching lie, as one would naturally expect, on the border line between failure and success. One of these is related to the "project."

The early printed arithmetics had few problems compared with our modern books, and they had no abstract drill in computation. The instructor was supposed to prepare such drill as was needed, while the book supplied the problems. These exercises were not, however, like most of those to be found in our later books; they set forth a real situation which the boy going into trade must meet. They were often long and they suggested, although they were not precisely like, what is now sometimes spoken of as the "project." This word was very likely suggested by the one so long used by students of the great art school of Paris, and because of its novelty in America it has attracted considerable attention. There has been no generally accepted definition of the term, although there have been various books and articles written upon it. To some it means the keeping of actual accounts in a home or in respect to a garden or to the

building of a boat, relating usually to matters that are outside the school and that ordinarily involve no arithmetic not learned long before the "project" is assigned to the pupil; and therefore, from the standpoint of facility in computation, it is merely an exercise in economics, with no special mathematical value. This phase of the "project" seems to have arisen from the mistaken idea that "narrative problems" or "related problems" represent the common way in which we use arithmetic and are somehow more interesting than the isolated problem. The general purpose seems at first to have been to give to the pupil a real or supposed piece of work, say the furnishing of a house (letting him find the net cost and write out a bill of particulars), or the cultivation of a piece of land for a season, or the constructing of a radio outfit. This is a valuable thing to do if the boy is actually furnishing a house, cultivating the land, or building the apparatus, but is quite as fictitious as "partnership involving time" (to take an obsolete illustration) unless this is the case. It is usually said, in reply to such a criticism, that any enthusiastic teacher can make a fictitious project interesting; this of course is true, but it is equally true that such a teacher can make alligation, cube root, or any other topic, even if no longer found in current arithmetics, quite as interesting and probably fully as valuable.

The advocates of the project method have run the usual course, the extremists dividing projects into various types, each with its pedantic name; and the result is that the good which the method has is lost in the rather unfortunate outcome of their wasted efforts.

As stated above, the project is valuable chiefly as it represents a condition that is real to each child. Since this reality is difficult to obtain for a whole class, the method (in the form above mentioned) becomes largely

an individual one. All that a textbook can do for this kind of project is to suggest occasional types; if it goes farther, the long problem (for that is what it is, however it may be broken up or whatever name we give it) becomes tedious and time-consuming. So we have, in a recent arithmetic, one of these projects on gardening occupying the first seven pages of the book and upon a subject that means nothing to the great mass of city boys and girls, little to the girl in any part of the country, and little to either boy or girl even in agricultural localities. Such a project, carried to such a length, is specious, but it is not real nor is it interesting. People rarely use their arithmetic in that manner. Substantially all our uses of the subject are found in brief problems involving some isolated computation in economics, such as finding costs, checking bills, and balancing accounts. The project of the kind mentioned may well be introduced now and then, briefly, as a model for occasional individual or group work on some real situation; but the extreme form is not sound educationally, it fails to arouse or to maintain interest, and it is not representative of the actual uses of the subject. Even the authors of the work referred to evidently believed this, for the subject was born and buried in those seven pages and never appeared again in the book.

There is probably no other such time-wasting method of teaching arithmetic today as the extreme of the project method of the type above described, and yet there is probably no better means of showing the uses of arithmetic than by an occasional real project of this kind, actually worked out in the garden, on the farm, in the sewing room, or, preferably, in the class. At the worst the method is not arithmetic, but, as already stated, it sinks into very dull economics; at the best it is a stimulus to the individual to whom it is real. When

employed it requires much skill on the part of an enthusiastic, resourceful teacher and is seen at its best in a village community where it can be adapted to some local industry that is familiar to all,—in particular, say, to a series of individual plots in a school garden. Without such favorable conditions it is of doubtful value, and, in any case, before it is undertaken the expenditure of time should clearly appear to be worth the arithmetical gain.

A number of writers have suggested lists of problems and projects that are sociological, or vocational, or home informational; but they are rarely good arithmetical problems, because they are usually so poorly graded as to difficulty that they seem to the pupil unworthy his serious attention. In the computation of interest such problems or projects as show the danger of dealing with the "loan shark" found in the poorer quarters of our cities are valuable for a city child; they may easily be graded properly and they have high sociological significance; but in the upper grades, except for review and drill, projects involving nothing more than the adding of grocery lists are poor substitutes for those of mathematical difficulty commensurate with the pupil's powers.

It should also be said that projects on a subject like bookbinding, or cabinetwork, or carpet-weaving, have come to be considered quite as unreal as those on apothecaries' measures, except where the child is, at the precise time when they are given, engaged in these particular industries. The place for such problems, in any considerable number, is now properly held to be in the classes in bookbinding, woodwork, and textiles, where these exist, but not in the class in arithmetic. Projects from the general fields of industry, informational and well graded, are reasonably safe; but those from narrow fields that are known to but few children are receiving less and less recognition in the schoolroom.

Another mistake that was made a few years ago in the endeavor to make problems real was that of being ultratechnical. A business man once said to the writer that we ought to put into some of our arithmetics a certain feature of foreign exchange that was then and still is used by large banking houses in New York. But upon the suggestion that not one teacher in ten thousand would or should understand it, that not one pupil in a million would ever have occasion to use it, and that only bankers and mercantile houses engaged in large transactions in foreign exchange paid any attention to it, the supposed value of the scheme disappeared at once. So it is with many industrial problems; they are as technical as those involving troy weight, and quite as uninteresting and useless. There is a fairly good industrial field in which the teacher and textbook writer may work, but each must be on his guard lest the problems or the projects proposed be barren of good results.

Still another danger deserves mention. The excellent work done in certain industrial schools tempts the visitor to try experiments in the general public schools that are certain to yield no results that are satisfactory. Take, for example, such a school as that so successfully conducted by one of the great electric manufacturing companies. Here is a group of boys who are interested in a special trade. Everything that relates to that trade appeals to their imagination and interests. The diameters of wires for certain currents, the density of copper, the weight of a casting,—these and similar topics have an immediate interest for the entire group. Arithmetic does not need to search for projects under such conditions; they are all about and arise without effort. For the teacher of such a class, himself a mechanic, the teaching of arithmetic is a comparatively simple matter, just as it was in the old apprentice schools of which

these schools are a survival. It is feasible and natural to teach the subject under circumstances like these, just as it has always been taught in schools of this character, by means of concrete problems and extended projects, possibly followed by a little practice with abstract numbers. But to transfer this to the public school, with forty pupils in a class, girls as well as boys, representing widely different surroundings and interests, and taught by one who will remain, on the average, only four or five years in the profession,—to make this transfer is by no means simple; indeed, it is clearly impracticable, as our most successful teachers have found.

There has developed out of all the talk about projects, however, a considerable amount of good. It has led schools to see that, in its essence, it represents the spirit of such teachers and thinkers as Pestalozzi and Rousseau. As my colleague, Professor Hosic, has so skillfully pointed out, it suggests that "children are to learn through purposeful activities. Their experiences are to be complete, to include planning and judging results as well as executing, and they are to practice cooperation, team-play, working together. . . . In the psychology of active or dynamic experience and the principles of democratic group cooperation are to be found all the essential tenets of the doctrine. . . . Some teachers have wasted time on useless 'projects.' Others have failed to help their pupils to 'follow through' and organize for permanent retention the results of their study and practice. But no theory is a substitute for common sense."

This is a sane view of the whole matter. The project is, in one sense, not a thing, it is a mode of thought; it is worthy so long as it means reality and so long as it stands for natural, democratic group action. Because the medieval monk studied by himself in a quiet cell is no reason why a child should be compelled to study

his problems in arithmetic by himself, depending solely upon his individual initiative, when group activity is feasible and offers training in the very thing he will meet in life. When mere computation is involved, quiet isolation represents the ideal condition for the child and the adult alike; but in solving a problem or a set of problems, and particularly a set of related problems, group activity may well be brought to the aid of arithmetic.

The purpose of this kind of cooperation on a type of problem that represents real conditions is not to make the subject weak or flabby; it is to make it stronger through first making it both natural and interesting. That this can be done is evidenced by the fact that in all studies that have been scientifically made with respect to the likes and dislikes felt by students, mathematics in general and arithmetic in particular always stand among the favorite subjects,—an appreciation that is due, no doubt, to its precision and to the love for truth that characterizes the heart of youth and the hearts of all who retain the best that youth has to offer.

12. METHOD

One of the most hopeful signs of the last few years is the diminished attention given to any single narrow method. Instead of seeing only such devices as the Pestalozzi Method or the Grube Method, as was often the case a generation ago, teachers and textbook writers have now come to realize that one of the easiest things in the teaching of arithmetic is the creation of "method"—and one of the most useless. We may start off with the idea that all number is measure, and hence that arithmetic must consist of measuring everything in sight, and we have a "measuring method." It will be a narrow idea; we shall neglect much that is important; but if we

put energy back of it we shall attract attention and shall very likely turn out better computers than a poor teacher who is wise enough to have no method in this narrow sense of the term. Again, we may say that every number is a fraction, the numerator being an integral multiple of the denominator in the case of whole numbers. From this assumption we may proceed to teach arithmetic only as the science of fractions. It will be difficult, but, given enough energy and patience and skill, the children will survive it and will learn more of arithmetic than may be the case with listless teaching on a better plan. We might also start with the idea that every lesson should be a unit, and that in it should come every process of arithmetic, so far as this is possible, and we could stir up a good deal of interest in our "unit method." Or, again, we could begin with the idea that all action demands reaction, and that every lesson containing addition should also contain subtraction; that $6 + 4 = 10$ should be followed by $10 - 6 = 4$ and $10 - 4 = 6$, and that $2 \times 5 = 10$ should be followed by $10 \div 2 = 5$ and $10 \div 5 = 2$. By sufficient ingenuity a very taking scheme could be evolved, and the "inverse method" would begin to make a brief stir in the world. We might label the teaching which Plato suggested as the "play method," using games very freely, perhaps even neglecting the number work unduly in our desire to interest the child, and, again, we might thereby create the impression that we had evolved something new. This in fact has been the genesis, rise, and decline of methods: given a strong but narrow-minded personality, with some little idea such as those above mentioned; this idea is exploited as a panacea; it creates some little stir in circles more or less local; it is tried in a greater or less number of schools; the author and his pupils die, and in due time the method is remembered, if at all, only by some inscrip-

tion in those interesting pedagogical graveyards known to the world as histories of education.

The object in writing thus is manifest. For the teacher with but little experience it may suggest that there is no method that will lead to easy victory in the teaching of arithmetic. There are a few great principles that may well be taken to heart, but any single narrow plan and any single line of material must be looked upon with suspicion. Certain of the general principles of Pestalozzi are eternal, and are only today coming to be appreciated at their full worth, but the "ratio method" and the "measuring method" and the Herbartian steps are substantially forgotten.

Since this is not an essay upon method, but rather a statement as to the changes in sentiment as set forth in the textbooks of the last quarter of a century, no discussion of the current attempts at improvement in the details of instruction is to be expected. In a general way, however, it may be said that numerous valuable suggestions, made both by the theorist and by the practical teacher, are being tried out in the spirit which has been mentioned as characterizing the modern school. These cover the psychological experiments made with respect to the learning process, the study of the troublesome groups in number combinations, the simplest methods of approach to subjects like long division and the various operations with fractions, the method of study, and the economical conduct of a recitation. While all such topics are old, there are constant improvements in the spirit of their treatment, in the early steps in the learning process, and in a proper estimate of values of the topics taught. These improvements show a notable advance over the spirit, if not over the achievements, of the close of the nineteenth century, and their value is fully recognized in our most successful textbooks.

13. OBLIGATIONS AND HOPE OF EXPERIMENTAL SCHOOLS

It is proper at this time to say a word as to what the schools of observation and practice should stand for in these matters of method and purpose, since each enters into the making of our textbooks. It would be easy for such schools to concentrate upon some single point, some device of teaching, some particular line of problems, and to carry the work to an extreme that would attract attention and produce results that would be remarked upon. An investigation of some of this work done at the present time is interesting, particularly with reference to the tendency to pedantry of language already mentioned, but less with reference to any features that are particularly novel. The tendency to encourage experiments which tend to the sensational is a temptation to supervisors of experimental work. In the highest class of experimental schools the danger of such work is slight; the teachers are very skillful; the supervisors know how far it is safe to go; and the entire staff is made up of experts in the measurement of pupils' achievements. Schools of this kind are the sources for ideas leading to future improvement both in teaching and in the making of textbooks. But for the less highly developed training schools is the policy wise? They are established to train teachers to well-balanced class work, not to be extremists in experiment to the neglect of essential features of education. The graduates of such schools should know the best that there is in the leading theories of education, but they should also avoid the doubtful as well as the worst. They should feel that the prime desideratum in arithmetic is the ability to work accurately, with reasonable rapidity and with interest, and to know how to apply numbers to the ordinary affairs of life.

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They should also realize that to secure accuracy alone, to secure speed alone, to have arithmetic mere play without accuracy and a fair degree of speed, or to know how to apply numbers to life in only a slovenly way, even if the pupils keep records and make graphs of their progress,—these are extremes to be avoided at any cost, including the tempting cost of sensationalism. It is the mission of the training school to make the earnest, well-balanced teacher, first of all. With this duty goes the laudable one of making reasonable experiment and of trying out well-considered suggestions; but elementary training schools must at all hazards guard against the mistake of having it appear that an experiment is an accomplished result, or of sacrificing our children in unnecessary clinical work that is in many cases certain to fail. A resolution of the National Educational Association, a few years ago, contained the following recommendation:

The principles of sound and accurate training are as fixed as natural laws and should be insistently followed. Ill-considered experiments and indiscriminate methodizing should be abandoned, and attention devoted to the persevering and continuous drill necessary for accurate and efficient training.

It seems, from a survey of the schools at the present time, that this recommendation had its effect in spite of the protest of "reactionary" that it raised in certain quarters at the time it was made.

In most of our training schools there has for a long time been a tendency away from what is called the direct method of imparting number facts and toward the "rationalizing" process. This means that, instead of stating to a class that $4 + 5 = 9$ and drilling upon this and similar relations, the schools of observation have

generally tended to have the pupils discover the fact and then, through drill, memorize it until the statement of the result becomes mechanical. Experience shows that this tendency is a healthy one. A child likes to be a discoverer, to find out for himself how to add and multiply, always under the guidance of the teacher, and to see how to solve a problem before he knows of any dominating rule. It is when the teacher decides that the child is never to be told anything, never to be helped over difficulties, and never to be shown a short path before habit has determined upon a long one, that the danger of the rationalizing process appears.

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It is such lessons that the training schools can pass on to the younger class of teachers, and that they actually are passing on with great success. It is in the larger and best-equipped experimental schools that the more elaborate work of studying the learning process must be carried on, together with the standardizing of efficiency methods by the aid of control groups and of expert mathematical and psychological assistants.

14. THE GREAT ART OF TEACHING

A little earlier in this chapter a quotation was given from the writings of a man who invariably sought to speak the truth, and who usually succeeded,—John Ruskin. He never wrote upon the art of teaching arithmetic or of writing textbooks, but so broad were his mind and his human sympathies that he said many things that apply to elementary education in general as well as to the arts of the painter and the sculptor. He sought to lead teachers to see the great things of life and to realize more fully the nobility of their profession, and in doing so he spoke to the teacher of arithmetic as distinctly as to the teacher of any other branch of knowledge. One of his addresses contains the following expression relat-

ing to the spirit which inspires a great teacher or a successful maker of textbooks:

To create anything in reality is to put life into it. A poet, or creator [the words originally meant the same], is therefore a person who puts things together, not as a watchmaker steel, or a shoemaker leather, but who puts life into them. His work is essentially this: it is the gathering and arranging of material by imagination, so as to have in it at last the harmony or helpfulness of life, and the passion or emotion of life.

What more inspiring picture could be painted of the ideal teacher or writer who successfully combines the best that the last few years have emphasized with respect to arithmetic?

The difference between a good course of study and a bad one, between an ideal and a commonplace sequence of topics, between an inspiring and a dull teacher, between a great textbook and one which sees only the trivial things of life, between a pedant and one who clothes the truth in simple phrases,—all this, too, is brought out by Ruskin with his usual clarity of thought. In his *Modern Painters* he has these inspiring words:

The difference between great and mean art lies, not in definable methods of handling, or styles of representation, or choices of subjects, but wholly in the nobleness of the end to which the effort of the painter is addressed. We cannot say that a painter is great because he paints boldly, or paints delicately; because he generalizes or particularizes; because he loves detail, or because he disdains it. He is great if, by any of these means, he has laid open noble truths, or aroused noble emotions.

So with the teaching of mathematics, our work is great in the classroom if we feel the nobility of that work, if we love the human souls we work with more than the

division of fractions, if we love our subject so much that we make our pupils love it, and if we remember that our duty to the world is to help fix in the minds of our pupils the facts of number that they must have in after life. If we make our class periods mere hours of aimless play, we shall not have "laid open noble truths or aroused noble emotions," nor shall we have imparted with joy the information that the child must acquire. It is in such a spirit as Ruskin describes that we may help in perpetuating the great art of teaching. Probably the chief defect of our schools of education today is in the fact that they exalt the science of education and ignore unduly the great art of teaching. Against the progress of this tendency the teacher of arithmetic, as of other subjects, must always be on guard, and in this he has had the support of our leading textbooks for the past twenty years.

III

THE TEXTBOOK

1. PURPOSE OF A TEXTBOOK

The Textbook In the period under consideration there has been a healthy change in the accepted purpose of the textbook in arithmetic. At the close of the nineteenth century the long list of definitions, the formal explanation of processes, and the rule to be memorized for every operation were still largely in evidence. The purpose of the textbook was still held to be that it should play the rôle of teacher.

With the growth of training schools and the noteworthy improvement of the teaching force of the country the textbook has tended more and more to occupy its proper place as a helpful tool, as a servant instead of a master of the teacher. Our best teachers now feel free to omit exercises or explanatory matter when by so doing they can, to their thinking, improve upon the work. They supplement freely where the interest can thereby be increased, they borrow from other sources, and, in general, they use the book as a source of supply for well-graded problems relating to the practical life of today, for drill work, and for helpful suggestions as to sequence and presentation of theory.

This reform has been effected gradually, but if one compares the teacher's attitude with reference to the textbook today with that of a generation ago, he will see that the purpose of the book has changed in the es-

timation of the school, and has changed for the better. *The*
More and more the selection of the books is being placed *Textbook*
where it belongs, in the hands of the teachers who are
to use them. They know best the tools with which they
can accomplish their work most effectively, and they are
certainly better able to make the selection than a group
which does not see the question from the standpoint of
the classroom.

This being the case, it is incumbent upon teachers to
choose the best tools that their careful judgment may
designate, and it may therefore be helpful to call atten-
tion to some of the objectionable features still seen in
our arithmetics, and also to some of the features that go
to make up a usable book.

2. AMATEUR TEXTBOOK WRITERS

The convenient French word "amateur" meant origi-
nally a lover. We use it in English to designate one
who likes a subject but does not know much about it.
We have amateurs in music, in painting, in writing, in
politics, in education, and in every walk of life, including
the one which leads to textbook-making. To outgrow
the amateur stage in any of these lines requires much
time, more patience, and far more labor than the casual
observer can be expected to appreciate. It is impossible
to tell anyone how to write a textbook, just as it is im-
possible to tell him how to paint a great picture. Sug-
gestions can be made and errors can be pointed out, but
the successful textbook writer must, like the poet, be
born.

This may be illustrated by considering various cases
of arithmetics written by those who have succeeded in
other lines of education, some of them being genuine
leaders in their special fields, but who lack entirely the
understanding of the first principles of textbook writing.

Opening one of the books of such a writer at random, and turning the pages casually, we find one evidence of the amateur's work in the fact that many of the problems are not numbered, although they are intended for written work. The teacher cannot ask for the result of number 7, for example, but must ask for the result of the problem which reads "Find the discount on" etc. It seemed a small matter to the author, and he may have defended his action on the ground that the actual problems we meet in daily life are not numbered. To the teacher and the class, however, this one defect throws the book out of the conveniently usable class.

In another book of this type there are no chapters or divisions of any kind, the idea being that such features are artificial. To this extent the book is like a dictionary in plan, and it may facetiously be argued that there is no more reason for chapters in an arithmetic than there is in Webster, all of which would be true if the two books were to be put to the same use. The world, however, is the best psychological laboratory, and the world finds that there is pleasure in having marks of achievement, milestones of progress, or, to use a better figure, resting places along the route, where the traveler may pause and feel that this stage of the journey has been passed and that something new will now vary the monotony of travel. So it is with a book; unless it be a work which is not to be read, like a logarithmic table or a dictionary, the chapter division, by whatever name it may be called, is a necessity if we are to maintain interest in achievement and are to have resting places by which to mark our progress.

Another work of the same class is filled with expressions like \$.80, thus inculcating a habit that every business man or scientist would condemn, the business form being \$0.80, $\$ \frac{80}{100}$, or 80 ¢, and the corresponding form

used by the scientist being 0.8 or $\frac{4}{5}$. The form \$.80 requires only the erasure or omission of a dot, or the failure of the eye to see it, to change the sum of \$0.80 to \$80, and the business world takes no such chances. *The Textbook*

We likewise have such statements as these: "Percent of" means "hundredths *times*," but "*what* percent of" means "how many hundredths *of*,"—which is at least an interesting problem in English.

The following is also a type of the statements usually found in books of this kind. After speaking of "*hundredths* or *percent*," the two terms being given as synonymous, the author of one of these works goes on to speak of "changing from *hundredths* with the decimal point to *percents* without the decimal point," as if the two terms had suddenly become different.

One book of this type says that "any quantity is a multiple of some unit," but the pupil and teacher will search in vain in the book in question for any definition or explanation of either "multiple" or "unit." Is 1, for example, a unit? Is $\sqrt{2}$ a quantity? Of what unit is $\sqrt{2}$ a multiple?

In the sentence "The lengths of corresponding lines in similar triangles are in the same proportions," what is meant by "proportions"? Does it mean "ratios"? and if so, why the plural form?

Such books measure the heights of children to one *millimeter*, and they give problems involving the addition of 3.0168 mi., 292.5 yd., and 6800 in., and yet claim to be practical and modern. They are usually poorly printed, their pages look bad, their tables and rules are often read only after considerable eyestrain, they freely use terms that are neither defined nor explained, their indexes are so incomplete as to be of no help, and they introduce applications that represent conditions entirely foreign to the pupils' interest, needs, and knowledge.

They frequently require the pupil to turn a leaf in the midst of the discussion of a problem; they give a picture of a cord of wood that is chiefly made up of a man and two fractious horses, they make the myriameter (which is never used) of equal importance with the meter and millimeter, they fail to recognize that proportion is obsolete as a means of solving interest problems, and in general they are reactionary in fact while trying to be progressive in aspirations. Such books are of value only in that they are often written with the best of intentions; but in actual accomplishment, in the use of modern methods, in the recognition of the broad outlook of psychology in contrast with the narrowest kind of view with respect to a few small features, they are not successful. The history of education, like the history of art or of industry, is filled with the record of the failure of the amateur to whose mind all things have equal weight, to whom the infinitesimal seems as important as the finite, and who works without plan and prides himself upon so doing. With the amateurish book of this kind the schools of today have troubles, although not for long, the books being soon discarded. The troubles exist, however, and they come about because of the natural desire of ambitious teachers to seek the new in the hope that they may thereby obtain the good,—a desire that is so often doomed to disappointment.

3. UNBUSINESSLIKE NOTIONS

In considering the textbooks of today it is also desirable to call attention to the fact that not infrequently they set forth what may be called unbusinesslike notions, and thus sow the seeds of future trouble.

In some respects the modern school resembles the cloister schools of the Middle Ages, although we are

happily breaking away from many of the ancient ideals and customs. The most noteworthy remaining evidence of this resemblance is seen in the barrier that is often erected between the business and the academic world. The school tends to be a law unto itself, with altogether too little care for the world beyond its walls. Thus we use the word "difference" where the world uses "rest" (as the school did at one time) or "balance"; we say "subtract" where the world says "deduct"; we use "dividend" to mean a number which we divide, but the world uses it to mean the income from stocks; we use such terms as "least common multiple," which are forgotten as soon as we leave school; we use $\div$ as a sign of division, which is rarely found in business; and so on through a long list. We who teach feel that the world should adopt our language and methods, while the world does not give the idea even the slightest attention, to say nothing of considering any idea of adoption.

This is unfortunate because the child has to unlearn much that he is taught in school, and many things which to the teacher seem extremely important do not secure even the passing attention of the world at large.

It is difficult to change some of these things, so strong is tradition and habit. We do not use the word "multiplicand" so often that there would be any inconvenience in saying "number to be multiplied" instead. The European world commonly did just this (when speaking Latin) until relatively modern times, and got on very easily. It would probably be simpler in beginners' classes to use the single term "result" than to require the child to distinguish among "sum," "difference," "product," and "quotient." The word "product" commonly served for the result of addition until recently, and the world had no difficulty. But our habits are fixed and the change will come slowly; indeed, the matter is

The not particularly serious, even though a considerable pro-
Textbook portion of people will admit that they do not know which is the minuend and which the subtrahend in subtraction—and there is no good reason why they should.

But though the schools teach many terms and even special processes in arithmetic that are not found in the business world, this is no reason why they should attempt to make matters worse than they are. This would seem to be axiomatic, a mere common-sense way of looking at things. And yet, in spite of all this, we find teachers not infrequently giving such forms in “short division” as

$$\begin{array}{r} 25 \\ 5 \overline{)125} \end{array}$$

—a form which the business world never uses and probably never will use. It is one of the unbusinesslike notions that come up in the schools continually, supported by an argument which seems plausible enough unless one stops to think about it. The argument is that we write the result above the number divided in the case of “long division.” There is one and only one reason for doing this; namely, that we make fewer mistakes in placing a decimal point if we proceed in this way, and there is no vacant place below the number divided where the result can be written. It is argued that, because we place the result above in “long division,” the child will be confused if we place it below in “short division.” This is somewhat, although not precisely, like arguing that the result in addition should go to the top. This, indeed, was once the custom, and there is something to be said in its favor, since (partly because the pencil is in our way if we point to the figures as we add downward) we usually add upward, whence the phrase “to add up.”

The fact is, the world refuses to place the result in “short division” above the number divided, and it does

so for the very good reason that it is not so easy for a computer. It is a common occurrence that we wish at once to perform another operation on a quotient, as when we add a number to it, subtract one from it, multiply it, or again divide, all of which processes require us to copy the number if it is written above, with the attendant chance of error. The fact that we work down a page as we write, and the further fact that we write all our other results below whenever reasonably convenient, are added reasons why the world will simply smile, if it even takes the trouble to do that much, at this unbusinesslike notion of a relatively small number of schools.

We find in our schools many other similar notions (for they can hardly be dignified by any other term),— notions in which the future needs of the child are forgotten in our desire to assert the authority of the teacher. Thus we have schools which use such clumsy devices as the one here shown for the addition of $2\frac{1}{2}$, $5\frac{2}{3}$, and $3\frac{7}{8}$.

This is varied by different teachers, but in any case such a complicated form is inexcusable; it is not businesslike, it is notional, and it is an evidence of very poor teaching.

In the first place, the chance of adding $2\frac{1}{2}$ to $5\frac{2}{3}$ and $3\frac{7}{8}$ in a practical business problem is rather remote; but if it is met at all, there is the simple, straightforward way of adding that the commercial world uses, and it has none of the difficulties illustrated above.

It is a very good rule to work in business matters as the business world works, to use the terms that it uses, and so to fix the pupil's habits and mechanical "reactions" that he will not have to unlearn a variety of things before he can make any progress in his vocation.

$$\begin{array}{r}
 24 \\
 \hline
 12 \\
 16 \\
 21 \\
 \hline
 10
 \end{array}
 \quad
 \begin{array}{r}
 24)49(2 \\
 \underline{48} \\
 1
 \end{array}$$

$$10 + 2\frac{1}{4} = 12\frac{1}{4}$$

The Textbook To the extent that any textbook fails to embody this spirit it is a failure. The last two or three decades have seen many changes in business customs, and in general the textbooks have kept pace with these steps toward business efficiency; but it is still evident that too often there is a failure to grasp the idea that the business world dictates business customs to the school, not the school to the world.

4. GENERAL TYPES OF TEXTBOOKS

In general, in this country, arithmetics now appear in three forms: (1) a two-book series, the first book being intended for grades I-IV (usually grades II, III, and IV), and the second for grades V-VIII; (2) a three-book series, intended respectively for grades II-IV, V and VI, and VII and VIII; (3) a series by grades. Where there are junior high schools with modern courses the pupils take up special books in the first year, that is, in grade VII.

Formerly, say a century or more ago, it was the custom to study arithmetic from a single book, after the boy (for the girl, as already stated, seldom understood mathematics of any kind) could read and write. The child was mature enough to understand the subject after a fashion, and he "went through" the book. But as arithmetic began to work its way down to the earliest grades it was found impracticable to follow this plan, the subject as so taught being too difficult for young minds. The ordinary textbook was therefore preceded by a primer on arithmetic, and thus a two-book series was formed. From this beginning numerous experiments have proceeded, the result being, at present, the three types above mentioned.

The curriculum prescribed by the best of these textbooks has been the result of the experience of teachers.

In every city, in several states, and by numerous associations, courses of study have been arranged setting forth the material that experience has shown can be used to advantage in the several school years, and selections have been made from the current arithmetics to supply what was needed. In other words, the practical teachers of America arranged their own books to a great extent, basing their selection of material upon an empirical psychology and giving to the child what his interests and capabilities suggested.

It is out of such a movement, spontaneous but thoroughly sound, that the later American textbooks have arisen, and the care and earnestness given to their preparation by various authors and publishers should have the commendation of all.

These books are and probably will continue to be of two distinct types, each with certain peculiar merits and each capable of producing good work. First there is the topical arithmetic, that is, a book arranged by topics, a subject like percentage being taken once for all, the pupil studying it until it is thoroughly mastered. Such a book has two great merits: it tends to keep the child upon each subject long enough to give him a feeling of mastery that he would not have if he studied some of the scrappy books constructed on the extreme spiral system; and it allows a teacher who wishes to adopt a moderate spiral to do this in a manner that will meet the local conditions. By this we mean that some classes need more work in a subject like simple interest, when it is first taken up, than others do. A teacher may therefore select as much or as little as is necessary when a topic is taken up for the first or the second time, and this is perhaps more easily done from a topical than from another form of book. A teacher or a school may also easily adjust books of this type to meet the de-

The mands of new or peculiar courses of study. It is always
Textbook legitimate to omit topics, or to make such moderate rearrangement as may be demanded by the course of study of any particular locality, and changes of this kind should freely be made. On the other hand, it is unfortunate that so many different courses should be adopted from year to year, only to be discarded as soon as there is a change in administration. While it is only by experiment that we make progress, no one can carefully examine the dozens of new courses that are issued every year without seeing that they often show evidence of undue haste and of the desire for mere change. To meet this restlessness the topical arithmetic is probably the better type, for it is more readily adjustable to all kinds of courses, and it has proved its usability with all sorts of teachers and pupils.

The second general type of textbook is one that attempts to fit the course of study in a large number of schools. In general these books agree in a number of particulars: (1) certain subjects, such as the most commonly used operations, appear several times in the course; (2) others, such as the business application of simple interest, appear perhaps one or two times, with slightly increasing difficulty; (3) still others, like circle measure, may be sufficiently treated once for all; (4) the closing year of arithmetic is devoted to a study of such higher problems of business as can be understood by the children. These are some of the articles of agreement, and they go to show the common-sense principles on which our experimental courses of study and our textbooks in arithmetic are based.

As to which of these two types is the better it is impossible to give a general decision. It depends largely upon the school and the teacher. With the one book the teacher arranges the matter to suit the needs of the

class; in the other it is already arranged to suit the average pupil. Neither will fit each individual case, and the great thing is not so much the arrangement of the matter as the modern spirit displayed in the omission of the obsolete and the introduction of new, vital, interesting, intelligible problems of today. On this point great emphasis should be laid. A teacher should never make the mistake of feeling that everything in a textbook should be taken by the class. The best textbooks must give enough work for all needs, but this means that they must give too much for most needs. The teacher must judge when a class has reached the proper stage for taking up a new topic, and when that stage is reached the old topic should be dropped, whether all the drill work and applications have been taken or not. What is needed in one part of a city may not be necessary in another part, and an amount of drill work that may be commended in one community might well be condemned in another. One of the distinguishing features of the work in the best European schools is the freedom with which the teacher omits matter from the textbook and supplies problems of local significance.

In a general way it may be said that experience in recent years has shown that schools prefer a book so arranged as to be followed closely. There is less attempt to make courses that are purely local, it being recognized that a writer who examines and is familiar with the leading curricula in large cities and in the various states is quite as capable of preparing a proper sequence of topics as a local committee of teachers. While city pride may favor the purely local plan, there is less difference in courses and less insistence upon separate courses for each city than was formerly the case. Partly on this account the three-book series is at present more popular than the two-book series.

5. CHARACTERISTICS OF A USABLE ARITHMETIC

The

Textbook

The best modern textbook in arithmetic has many points of superiority over its predecessor of a generation ago. These points may be said to characterize a usable work and may be summarized as follows:

1. The matter is arranged according to the best American experience as to the work to be given in each of the several grades. If a book does not give such an arrangement, it does not rank in the usable class.

2. The problems are arranged with reasonable reference to difficulty. If problems for the upper grades contain merely the computations learned in the lower grades, this book, like the other, is barred from the usable class.

3. The topics of the book are also arranged with respect to difficulty, not merely as to calculation but also as to the complexity of the subject. For example, a difficult problem in electricity, if it has any place at all, should find that place only at the end of the book, and a problem involving the discounting of a note should be given after the mere writing of checks and deposit slips has been studied.

4. There are explanations sufficiently complete to assist the teacher in making a process or a typical problem sufficiently understood. Not to have this feature is to go back to the Middle Ages. Such explanations should be neither unnecessarily complicated nor too abridged. Simplicity of language should characterize all such work.

5. At every important step there is a fair amount of motive established, but a book that degenerates into little besides motive is of no value as a textbook, however suggestive it may be to a teacher who is searching for "motivation" material.

6. The book furnishes plenty of drill in abstract computation so as to allow the pupils to fix their habits of mechanical calculation. The teacher should not be called upon to supply such material; it is much easier to omit than to supply. *The Textbook*

7. There is proper attention to the visualization of the operation, column work largely predominating over such forms as $27 + 49 + 126$ and $4208 - 769$. Substantially all the pupils' computation should be arranged with the figures in columns instead of in horizontal lines, and the textbook should assist the teacher in this respect.

8. The customs of business are observed instead of the opinions of a limited number of teachers. It should be very rare for a textbook to find itself out of harmony with commercial usage. This is one reason why, as already stated, it is better in short division to write the quotient below the dividend, in spite of the well-known but futile argument of some teachers to the contrary.

9. The book limits its use of fractions chiefly to those found in actual practice at the present time. The difficult fractions of a quarter of a century ago have, in general, given way to this simpler type, and a textbook that fails to recognize this fact puts itself in the unusable list.

10. The subject of percentage, aside from its applications, has recently come to be looked upon as a phase of decimal fractions, and its presence as a long and distinct chapter, not tied up with decimals, is no longer felt to be desirable. With this has come a recognition of the fact that it is the direct case, say like that of 5% of \$150, that is the most important. Books that present any considerable number of "cases," and that unduly emphasize the indirect ones, can hardly expect to rank high in the usable class.

11. The modern book recognizes the value of ratio in commercial, industrial, and agricultural problems. But if a book uses proportion to solve commercial problems of the type of a generation ago, it has not the modern spirit.

12. A book can be judged fairly well as to its proper gradation by considering whether its early problems involve in general only one step, the two-step exercises being taken up later. This, however, manifestly does not exhaust the criteria on this point.

13. A fairly accurate judgment can be formed with respect to the author's appreciation of the psychological difficulties of a child by observing whether technical terms are unnecessarily used, this usage being in the direction of pedantry and of a failure to realize the necessity for simplicity of language. In some of our recent books this pedantry may be seen in the unnecessary use of technical language and symbols in the statements of the problems to be solved.

14. The problems of a usable book of the present day are so arranged as not to discourage the pupil by the adoption of an excessive narrative style, by the undue use of long projects, or by the dependence of one problem upon the answer to another, an error in the latter answer vitiating the subsequent results. Exceptions to this principle may properly be made for the sake of variety, but in general a book that fails to recognize the validity of the principle turns out to be unusable in the classroom.

15. A textbook may also be placed in the unusable class if its problems tend to confuse or puzzle the pupils through lack of clarity or through prolixity. Occasional problems may be set with more numbers than are needed for their solution, but the pupils should be told that this is to be done, so as not to puzzle them for the

mere pleasure of confusing them. The plea that this is the type of problem frequently met in daily life is not warranted by the facts. *The Textbook*

16. A book may be judged largely by its freedom from eccentricities. While always receptive to what is good, and welcoming the novel if it is also in this class, the modern book is coming to be judged more and more by the absence of those eccentricities which manifestly are intended to catch the popular fancy of the moment, but are sure not to be of permanent value. A recent writer helped put his book out of the usable class by giving an elaborate treatment of the income-tax blank of a certain year, when it was evident to everyone who saw it that the treatment was obsolete before the book was off the press. It is such eccentricities as this that place so many textbooks out of the race. Some books show this eccentricity in the excessive use of such numbers as \$0.347 and \$28.0286; for while such numbers have an important place in subjects like taxation, foreign exchange, and the investments of large corporations, they are not the numbers of daily life and hence should be used sparingly.

17. Among the chief points by which to judge a book is its first treatment of long division. If it approaches the subject by easy, well-graded steps, not attempting to hasten unduly, anticipating each difficulty that the child may encounter in this the most difficult operation that he has met, beginning with such simple divisors as 11, passing thence to 21, 31, and so on, then trying 22, then going to 29, and finally ending with two-figure numbers in general, the book stands a good chance of being usable; but otherwise it is quite certain to be in the opposite class.

The list might be extended to greater lengths, but it is already sufficient to show some of the points by which to

judge a modern arithmetic. This judgment can no more be made by counting exercises or pages or topics than we can judge the quality of a friend by any mechanical method of counting his good and questionable points. We like a book because it appeals to us, just as we like a companion, not because of any diagnostic scale. The points suggested, however, are among those which may have some interest to us, just as we may like to enumerate the desirable features of an individual whom we think worthy of our friendship; but they are never conclusive.

6. THE MECHANICAL EXCELLENCE OF A BOOK

We may admire a friend who never brushes his clothes, rarely wears clean shoes, or even fails in the ordinary demands of good breeding; but we certainly prefer that he should pay at least some attention to his appearance and his manners.

Similarly, we may endure a book that is badly printed, that has a slovenly appearance, and that has illustrations of the style of a generation ago; but we prefer it otherwise.

The modern American textbook, at its best, is the most attractive textbook in the world. More care is given to its mechanical excellence than is the case in any other country, and more attention is paid to adapting it to the comfort and general well-being of the child. A book that is not up to the best standard of mechanical excellence is, at least, to be viewed with suspicion.

The tests of a book on arithmetic, from the mechanical standpoint, should include the following details:

1. Has it large type of conventional design; that is, is the type of a kind that will not cause eyestrain when the pupil reads the pages? In particular, are the fractions of a proper size to be easily read? If not, the book

has no claim upon the attention of the school. To determine this matter several standard textbooks may be opened and placed side by side, and the eye allowed to pass from the pages of one to those of the other. *The Textbook*

2. Is the paper of good quality and without undue gloss which hurts the pupils' eyes? Is it strong so as not to tear easily?

3. Is every page enough of a unit so that the pupil never has to turn a leaf in the midst of a problem, a discussion, or an explanation? Does each page look well? Is it a model for the pupil in writing his own work? A comparison of pages in several books of high grade is desirable in answering these questions.

4. Are the illustrations such as will interest the pupils? Do they assist in understanding the spirit of the problems? Are they works of art, being models which shall influence the pupils to appreciate good drawing, or are they poorly planned and poorly executed? A comparison of several good textbooks, opened side by side, will also assist in deciding upon this important matter. Many textbooks have very poor illustrations, some of them being quite as bad as those of a generation ago. These aids to teaching arithmetic should also be aids in appreciating good art, and not be specimens of poor drawing, muddy in appearance, and caricatures in their representations of people and animals.

5. Is the binding of the book durable? Is the general color such as not to show soil easily, and are the colors of the decoration harmonious?

6. Has proper attention been paid to the pages of contents and index? Does the former give the plan of the book succinctly, clearly, and in language that both teachers and pupils can easily understand? Is the index sufficiently complete and is the typography up to the standard set by the best printers?

7. Taking the book as a whole, and considering it for this purpose from the mechanical side alone, is it the best-looking arithmetic among the modern publications? If so, there is a good probability that the same care that has been shown in those details that make for perfection in the manufacture of the book has also been shown in comparing the leading courses of study in the country; in weighing carefully the theories of the educational experts; in considering with equal care the conditions of the schools and the experience of practical teachers; in the application to the work of such psychological principles as give evidence of being useful in the classroom; in the use of common sense in avoiding the hysteria which leads to extremes and makes small matters seem overimportant to the amateur; and in the exercise of good judgment in making a textbook that will be a genuine help to teacher and pupil, offering abundant material for problem-solving and for drill, arranging this material in order of difficulty and presenting it in the most attractive manner possible.

Such are some of the points to be considered in judging a textbook from the mechanical side. In attaining to the present perfection of textbooks the great advance in manufacture in the last twenty-five years may be seen by comparing the books of a generation ago with the best that are produced today. In no other country has there been any such advance, for in Europe the war conditions, if nothing more, would be sufficient to account for the failure to keep pace with America. This advance has not, however, been confined to mechanical excellence; the care taken in planning and executing the best American arithmetics has certainly never been surpassed at any time or in any country. It is for this reason that our books rank today as the most usable ones to be found anywhere in the world.

PUBLISHERS' SUPPLEMENT
HISTORICAL PAGES IN ARITHMETIC

PUBLISHERS' NOTE

The following illustrations serve to supplement Professor Smith's essay on the Progress of Arithmetic in the Last Quarter of a Century. They give a visual picture of the development of arithmetic textbooks not only in the period mentioned but in some cases from the days of the early printed books. The illustrations from the ancient texts are given in Professor Smith's "History of Mathematics," Ginn and Company, Boston, 1923.

Incommincia oia practica molto bona et utile
a ciascaduno chi vuole vfare larte dela mercha-
dantia. chiamata vulgarmente larte de labbacho.



Regato piu e piu volte da alcuni
iouani a mi molto vilecassimi: li
quali pretendeuano a vouer volcr
fare la merchadantia: che per loro
amore me faceffe assidigarme v
no puochorde vargli in scritto qualche fundameto
cerca larte de arismettrica: chiamata vulgarmente
labbacho. Unde io constretto per amor di loro: et
enadio ad vilitati tuti chi pretendano a quella: se-
gondo la picola intelligentia del ingegno mio: ho
deliberato se non in tuto: in parte tane sanffare a
loro. acio che loro virtuosi desideri vule frutto re-
treuere posseno. In nome di dio adoneba: toglio
per principio mio el vito de algorifmo cosi viedo.

Ute quelle cose: che da la prima origine
hanno habuto producineto: per ratone de
numero sono sta formate. Et cosi come so-
no: bano da fir cognoscute. Pero ne la cognitione
de tutte le cose: questa practica e necessaria. E per
merar nel pposito mio: primo sapi lectore: che qu-
to fa al proposito nostro: Numero e una moltitu-
dine congregata onero infembrada da molte vni-
tade. et al meno da vo unitade. come e. 2. el quale
e lo primo e menoze numero: che se truoua. La v-
nitade e quella cosa: da la quale ogni cosa si vitta
vna. Segondario sapi: che se truoua numeri de tre
maniere. El primo se chiama numero semplice. lal-
tro numero articulo. El terzo se chiama numero

Lesson 4.

THE SIGN +.

The sign + stands for AND.

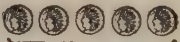
Complete these sentences, using the sign + :

7	2 = 9.	4	3 = 7.
6	3 = 9.	3	7 = 10.
5	4 = 9.	8	2 = 10.
5	3 = 8.	5	2 = 7.
6	2 = 8.	4	2 = 6.

Complete these sentences :

$3 + 2 = ?$	$4 + 2 = ?$	$4 + 3 = ?$	$6 + 3 = ?$
$5 + 2 = ?$	$6 + 2 = ?$	$7 + 3 = ?$	$3 + 4 = ?$
$7 + 2 = ?$	$8 + 2 = ?$	$5 + 3 = ?$	$5 + 4 = ?$

Tell quickly what number to put with /// to make eleven; with //// ////; with /// /// ///; with ////; with /// /// /; with /// ///; with ///; with /// ///.

How many eggs, at the rate of  can be bought for eleven cents? 

How many apples, , can be bought for eleven cents?

How many quarts and what part of a quart can you put into a can that holds eleven pints?

How many pints of syrup can you put into a pail that holds eleven gills, if there are three gills of syrup already in the pail?

THE GAME OF NIMBLE SQUIRREL

All work oral

1. The game we played to-day was Nimble Squirrel. The squirrel jumped from branch to branch and told the sums of the numbers. When he could not give the answers quickly he fell off. Be the nimble squirrel and name all the answers as quickly as you can, taking care not to fall off:



$$7+2 \quad 5+6$$

$$6+7 \quad 8+9$$

$$4+8 \quad 9+7$$

Beginning at the bottom of each column, tell these sums as you jump from branch to branch:

$$2. \ 5+2 \quad 3. \ 9+3 \quad 4. \ 8+4 \quad 5. \ 4+6 \quad 6. \ 5+8$$

$$3+2 \quad 6+3 \quad 6+4 \quad 8+5 \quad 7+6$$

$$8+2 \quad 2+3 \quad 4+4 \quad 5+3 \quad 3+4$$

$$6+2 \quad 8+3 \quad 7+4 \quad 4+2 \quad 5+5$$

$$9+2 \quad 4+3 \quad 5+4 \quad 7+1 \quad 3+9$$

$$7+2 \quad 5+3 \quad 9+4 \quad 3+8 \quad 4+7$$

DRY MEASURE.

Dry Measure is used in measuring dry articles, as grain, seeds, fruit, vegetables.

TABLE.

2 pints (pt.)	= 1 quart (qt.).
8 quarts	= 1 peck (pk.).
4 pecks	= 1 bushel (bu.).

Hence 1 bu. = 4 pk. = 32 qt.

NOTE 1. The gallon of liquid measure contains 231 cubic inches. Therefore the quart of liquid measure contains $57\frac{1}{2}$ cubic inches. The bushel of dry measure contains 2150.42 cubic inches. Therefore the quart of dry measure contains $67\frac{1}{2}$ cubic inches.

NOTE 2. In measuring grain, seeds, and small fruits, the measure must be *even* full. In measuring apples, potatoes, and other large articles, the measure must be *heaping* full.

1. Reduce 5 bu. 3 pk. 4 qt. to quarts.
2. Reduce 4056 pt. to higher denominations.
3. Multiply 7 bu. 2 pk. 7 qt. by 9.
4. Divide 25 bu. 3 pk. 2 qt. by 7.
5. How many 4-quart measures will 2 bu. 2 pk. 4 qt. fill?
6. Divide 20 bu. 2 pk. by 8.

Add:

7.		
bu.	pk.	qt.
5	1	3
3	3	3
7	2	7

8.		
bu.	pk.	qt.
8	3	1
9	3	7
9	3	6

9.		
bu.	pk.	qt.
121	1	7
156	3	6
132	3	5

Subtract:

10.		
bu.	pk.	qt.
5	2	2
3	1	7

11.		
bu.	pk.	qt.
8	1	2
4	3	3

12.		
bu.	pk.	qt.
150	2	5
136	3	7

A TYPICAL PAGE FROM A POPULAR ARITHMETIC IN USE ABOUT 1900

This should be compared with the model one which faces it

V. MEASURES AND FRACTIONS



MEASURING FRUIT AND GRAIN

1. Name some fruit that is sold by the quart.
2. Name some grain that is sold by the bushel.
3. Name the three measures in the picture.
4. Read and learn this table of dry measure:

2 pints (pt.) = 1 quart (qt.)

8 quarts = 1 peck (pk.)

4 pecks = 1 bushel (bu.)

5. What special name is given to a half of a quart?
6. What special name is given to an eighth of a peck?

Such work should be treated orally and made the basis of an impromptu class exercise. It is desirable to have the measures in class, where they can be seen and handled.

A TYPICAL PAGE FROM THE WENTWORTH-SMITH SCHOOL ARITHMETICS

A model illustration showing the value of pictures in giving a correct idea of the relative size of common measures

LONG DIVISION.

The process of Long Division is the same as that of Short Division, except that the work is written in full, and the quotient is written *over* the dividend.

Divide 31864 by 87.

The beginner will find it convenient to form a table of products of the divisor by the numbers 1, 2, 3, ..., as follows:

$1 \times 87 = 87$	$4 \times 87 = 348$	$7 \times 87 = 609$
$2 \times 87 = 174$	$5 \times 87 = 435$	$8 \times 87 = 696$
$3 \times 87 = 261$	$6 \times 87 = 522$	$9 \times 87 = 783$

As 87 is more than 31, it is necessary to take *three* figures of the dividend for the first partial dividend. Of the products in the table

$$\begin{array}{r}
 \text{OPERATION.} \\
 366 \\
 87 \overline{) 31864} \\
 \underline{261} \\
 576 \\
 \underline{522} \\
 544 \\
 \underline{522} \\
 22 \text{ rem.}
 \end{array}$$

that do not exceed 318, the greatest is 261; that is, 3×87 . Hence the first quotient figure is 3, and is written over the 8, the *right-hand figure* of the first partial dividend; then 261 is subtracted from 318. To the remainder 57, the next figure 6 of the dividend is annexed. Of the products that do not exceed 576, the greatest is 522; that is, 6×87 . Hence 6 is the next figure of the quotient, and the next remainder is 54, to which the 4 of the dividend is annexed. Of the products that do not ex-

ceed 544, the greatest is 522; that is, 6×87 . Hence the next figure of the quotient is 6, and the remainder 22. Therefore the quotient is 366, and the remainder 22.

After a little practice the operation of division can be performed without the aid of a table of products.

If at any step the product is greater than the partial dividend, the number denoted by the quotient-figure is too large and must be diminished; if the remainder is greater than the divisor, the number denoted by the quotient-figure is too small and must be increased.

THE FIRST PAGE IN LONG DIVISION AS PRESENTED ABOUT 1900
Notice the extreme formalism and the failure to approach the difficulty slowly

Long Division. If a dealer pays \$176 for 11 Boy Scout suits, how much does he pay for each suit?

For each suit he must pay $\$176 \div 11$, so we need to know how to divide \$176 by 11. In such division we need not write the dollar sign (\$).

The following shows the work which we do:

Divide, $17 \div 11$
 Multiply, 1×11
 Subtract; then $66 \div 11$
 Multiply, 6×11
 Subtract; no remainder.

WRITTEN WORK	CHECK
$\begin{array}{r} 16 \\ 11 \overline{)176} \\ \underline{11} \\ 66 \\ \underline{66} \end{array}$	$\begin{array}{r} 16 \\ 11 \\ \hline 176 \end{array}$

The teacher should put this work on the blackboard, explain what is done in each step, show the pupil what he is expected to write (Written Work), and explain the check. Although short division should be used in dividing by 11, this is the best divisor for the purpose of introducing long division. Tell the class the difference between short division and long division.

DIVIDING BY 11

Divide each of the following numbers by 11:

1. 132. 3. 1232. 5. 1342. 7. 4752. 9. 3641.
 2. 143. 4. 1243. 6. 1452. 8. 3542. 10. 7898.

11. At \$11 each, how many football suits can a dealer buy for \$726? for \$737? for \$847?

12. At \$11 each, how many Boy Scout suits can a dealer buy for \$253? for \$363? for \$374?

13. If 11 girls weigh 649 lb., find their average weight.

THE FOURTH PAGE IN LONG DIVISION FROM THE WENTWORTH-SMITH
 SCHOOL ARITHMETICS

The preceding pages have led up gradually to this succinct presentation

DRILL FOR SPEED

Add the following :

1.	1	3	1	1	5	4	6	7	2
	4	7	8	7	6	9	7	8	6
	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>
2.	2	1	4	6	1	3	5	6	8
	9	6	7	9	2	6	7	8	9
	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>
3.	2	4	2	4	7	2	3	3	6
	3	8	7	5	9	4	8	5	6
	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>
4.	5	3	1	5	8	1	3	9	1
	5	9	5	8	8	3	3	9	1
	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>
5.	1	4	2	2	2	3	5	7	4
	9	6	2	8	5	4	9	7	4
	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>

The ultimate achievement of each pupil in Grade III should be the ability to give the answers to the above additions orally in 30 seconds, the combinations being given in any order, left to right, right to left, up, or down. The sums of these forty-five combinations should be thoroughly learned. The record of each pupil should be kept, and the backward ones should receive special attention.

These forty-five combinations do not include the zero combinations or the reverse forms of $6 + 7$ as well as $7 + 6$, both of which psychology shows to be so important. On page 52 there is provided a time drill in all the combinations, the time allowance being eventually $1\frac{1}{4}$ min.

The table may also be used for individual practice and self-correction by transferring it to cards with openings through which the sums may be written. The correct answers may be written in the proper places on the back of the card, or the teacher may follow some other scheme for self-correction.

VALUABLE SUGGESTIONS TO TEACHERS

Suggestions, psychology, speed test, drill, all on a single page from the Wentworth-Smith School Arithmetics, Book One

SCHOOL-AND-HOME PROJECTS

1. In a certain school district the pupils take a school-and-home project; that is, they do a piece of work, keep track of expenses, and report the profit. Sometimes this work is gardening. The standard size of a garden plot for this purpose is 8 rd. long and 2 rd. wide. What part of an acre is this?

2. Sometimes a pupil cultivates a piece of land 10 rd. long and 4 rd. wide. What part of an acre is this?

3. Frank cultivates a garden 5 rd. long and $2\frac{1}{2}$ rd. wide. He puts a fence around it, using four wires nailed to posts. If a spool of 330 yd. of wire costs \$4, find the cost of the wire for the fence.

4. Richard Heath rented a piece of land from his father. He measured it and found it 39 ft. wide and 66 ft. long. He said that it contained a little over $5\frac{1}{2}$ sq. rd. Was he right? How much over $5\frac{1}{2}$ sq. rd. was there in the piece of land?

5. On the piece of land described in Ex. 4 Richard raised 29 pk. of potatoes. Find to the nearest thousandth of a bushel the amount of potatoes he raised to the square yard.

6. At the number of bushels to the square yard found in Ex. 5, how many bushels should Richard raise to an acre?

7. Richard sold the potatoes mentioned in Ex. 5 to his father at \$1.30 a bushel. How much money did he receive?

In many parts of the country the schools encourage school-and-home projects. These include school-and-community gardens, canning, poultry-raising, sewing, business, and cooking projects. Such projects encourage thrift, dignify labor, instill a love of nature, and furnish excellent practical applications of arithmetic. A reasonable number of projects are suggested in this book, but the complete carrying out of such projects often results in a waste of time and a loss of interest. Their use must depend on the needs of the school.

THE PROJECT IN THE WENTWORTH-SMITH SCHOOL ARITHMETICS

A common-sense view of the use of projects

IV. REVIEW AND DRILL

Nature and Purpose of Review and Drill. At the end of each chapter in this book there occurs a section like this one. The work is called Review and Drill, and it is optional with the teacher whether or not it shall be used or how much of it shall be taken. None of these sections is merely a review of the chapter immediately preceding it, for review work is already provided in the chapter itself; on the contrary, the review is general and covers many of the important processes in arithmetic that have been studied. This renders possible a continual return to fundamental and first principles, a constant brushing up in the essentials of arithmetic. This constant brushing up, drilling, and reviewing of the essentials is necessary for the attainment of the best results. The work is easily made interesting; it contributes to efficiency in computation; and it will, if carried out in the right spirit, help to remove the charge that most pupils are not quick and accurate in their calculations.

When the length of the school year permits, the study of these Review and Drill sections is recommended strongly; but the work being supplementary and optional, it doubtless will often be desirable for the teachers to select from these exercises only what is specially adapted to their immediate needs and to omit the rest. A certain flexibility is thus offered that will make the textbook adaptable to varying local requirements.

Each section consists of three parts: first, the Minimum Essentials, chiefly a drill in operations, emphasizing the *mechanical* side of arithmetic; second, an exercise in Problems without Numbers and Problems for Completion, emphasizing the *analytic* side of arithmetic; and, finally, live Industrial Problems that emphasize the *practical* side of arithmetic.

It will at once occur to every teacher that in these Review and Drill sections, with their threefold, inclusive emphasis, there is ready at hand a series of *efficiency tests* which will make it easy at frequent intervals to check up the work of the school as well as the work of the individual pupils.

NEW FEATURES IN THE WENTWORTH-SMITH SCHOOL ARITHMETICS

An illustration of the care to present the newer features to the teacher before the work is begun



AN-AM, BEL, AND THEIR SHEEP

ILLUSTRATION FROM PROFESSOR SMITH'S "NUMBER STORIES OF LONG AGO"

A book for supplementary reading in the grades



AS I was going to St. Ives
 I met a group of seven wives;
 Each wife had seven sacks,
 Each sack held seven cats,
 Each cat had seven kits.
 Kits, cats, sacks, and wives,
 How many were going to St. Ives?

7 seven  7 seven

I see 

I see 

A PAGE FROM THE WENTWORTH-SMITH "WORK AND PLAY WITH
 NUMBERS"

A modern textbook for Grade II

Parti 5349 > per 83

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534	
498	83
369	
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Parti $\frac{3}{8}$ p 60

Parti 13 > $\frac{1}{2}$ p 12

$\frac{3}{8}$ — 60

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13 > $\frac{1}{2} / \frac{1}{2}$

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Uienne 11 $\frac{1}{24}$

uienne $\frac{1}{60}$

Parti 60 p $\frac{3}{8}$

Parti $\frac{7}{8}$ p $\frac{7}{8}$

60 — $\frac{3}{8}$

$\frac{7}{8}$ — $\frac{7}{8}$

480

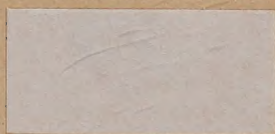
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uienne 160

Uienne o $\frac{1}{15}$



59